Inflation, default and sovereign debt: The role of denomination and ownership

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Abstract

Emerging market governments owe a mix of nominal and real debt both externally and domestically. This paper proposes a theory of sovereign borrowing with lack of commitment to characterize and quantify the implications of portfolio composition for debt, default, and inflation. It presents three findings: First, borrowing domestically, and especially in real terms, raises inflation. Second, ownership is more important than denomination for shaping macroeconomic outcomes. Third, nominal and external portfolios are welfare improving: They take advantage of the flexibility benefits of inflation without encouraging too much debt accumulation. We document empirical evidence supporting the testable model predictions. JEL: E52, E62, F34, H63

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1 Introduction

Sovereign debt crises in emerging market economies are often blamed on governments borrowing from abroad in real terms. With the debt burden fixed and foreigners holding the bonds, governments find it easy to default to reduce financing pressures in downturns. Having nominal debt at their disposal on the other hand is thought to be preferable in principle, but unaffordable in practice due to high inflation premia. The goal of this paper is to evaluate this reasoning. The question of what kind of debt structure is well-suited for crisis prevention is an important - and open - one for both national and supranational policymakers, and we contribute to the answer by characterizing and quantifying the role that debt ownership and denomination play in determining macroeconomic outcomes, default and inflation risk.

The framework of the analysis is a dynamic general equilibrium theory of optimal monetary and fiscal policy without commitment. The government issues non-contingent debt, prints money, raises taxes and can default outright on its debt in order to smooth residents' consumption against persistent productivity shocks. It issues a mix of nominal, real, domestic and external bonds, taking the portfolio structure as given. We study the optimal policy problem and how it depends on the composition of the debt.

Inflation and outright default are substitutes as both enable the government to relax its budget constraint: Inflation generates seigniorage revenue and - to the extent that it is unexpected and bonds are nominal - reduces the real debt burden, while outright default directly reduces the outstanding debt. But both are costly, which limits the extent to which the government optimally uses them: Domestic residents hold money because of a cash-in-advance constraint on consumption so inflation lowers their purchasing power and distorts their consumption decision. Outright default results in temporary productivity drops, bond market exclusion and lost savings for households if the debt is domestic. The government chooses between outright default and inflation by weighing the costs and benefits, so that higher shares of nominal relative to real debt tilt the optimal policy mix towards inflation and away from outright default.

Equilibrium inflation and default rates are determined as a result of the interaction of inflation and default incentives with optimal debt accumulation. On the one hand, inflation and default incentives are stronger when debt is high: Lower real debt burdens are more appealing in those states of the world because they enable the government to raise less revenue via distortionary labor taxes on households. On the other, high inflation and default risk raise borrowing costs and thus reduce debt accumulation in the first place. Bond portfolio characteristics that imply stronger incentives to inflate can therefore, in equilibrium, lead to higher or lower inflation, depending on how much the government optimally borrows.

We calibrate the model to Mexico using government debt data to match the observed debt portfolio composition as well as key statistics such as average inflation and default rates, and show that simulated data from the numerical solution of the model successfully reproduce other untargeted moments such as average seigniorage and tax rates, as well as the countercyclicality of inflation and net exports.

The calibrated model is then used to analyze the effects of shifts in portfolio composition. We present three main results: First, borrowing domestically drives up average inflation and default rates and implies higher debt to GDP ratios in the long run. When debt is owed to domestic households, the government faces weaker incentives to default and inflate since expropriation of residents entails negative wealth effects. In equilibrium, this force is strong enough to for average inflation and default rates to increase with the domestic debt share. Moreover, inflation increases as the real share of domestic debt rises. Despite the fact that inflation is a less useful tool to reduce the debt burden when debt is real, borrowing incentives are sufficiently strong and the tax base for inflation sufficiently small such that inflation rates rise as the government uses the printing press to generate revenue. Inflation decreases, on the other hand, as the real share of external debt rises because debt does not rise that much in equilibrium.

Second, we find that ownership is the more important determinant of equilibrium debt and inflation than denomination for a range of portfolios typically observed in the data. We compare model predictions for four portfolio types - largely external/nominal, external/real, domestic/nominal or domestic/real economies - and show that the two domestic debt economies are more similar to one another than the two nominal debt economies. Domestic ownership has a large and positive effect on inflation and debt to GDP ratios, while conditional on ownership, the effects of denomination are much smaller.

Both these results are testable implications of the models that we take to the data and find some empirical support for. In a panel of 24 emerging market countries for which government debt portfolio breakdowns are available, we show that inflation and debt to GDP ratios are positively associated with the share of real domestic debt, but not the share of real external debt, as implied by the model. We further show that inflation and debt to GDP ratios are more strongly correlated with ownership than denomination.

The third, normative result that we highlight is that nominal and external portfolios tend to be welfare-improving. These welfare effects of nominal debt are a result of its relative costs versus benefits: On the one hand, nominal debt provides flexibility as inflation can be used to incrementally adjust the debt burden, compared to the coarser tool of outright default. On the other hand, it entails distortionary costs that need to be weighed against the costs of default. The numerical results show that the flexibility benefits dominate. It is optimal to hold all domestic debt in nominal terms and the majority (around 75%) of external debt. For external debt, a non-zero real share helps reduce the temptation to inflate by reducing the usefulness of inflation as a tool to adjust debt burdens. The welfare effects of ownership depend on whether domestic ownership discourages expropriation sufficiently to make borrowing cheaper, or whether it facilitates so much debt accumulation that it actually drives up default and inflation in equilibrium. In the model, the latter effect dominates as domestic ownership provides strong incentives to accumulate debt, thus leading to welfare losses.

Literature This paper draws on two main strands of literature. First, it builds on the quantitative sovereign default literature following Arellano (2008) and Aguiar and Gopinath (2006) who study debt problems in small open endowment economies with incomplete markets and lack of commitment. There are a number of recent related contributions in this literature that study some of the aspects of this paper: The choice between nominal and real debt is analyzed in Engel and Park (2016) and Perez and Ottonello (2016) with no distinction between domestic and foreign creditors, and in Du et al. (2016) with no endogenous default. Du et al. (2016) show that countercyclical inflation emerges with nominal debt only if investors are risk averse, consistent with our results that inflation is more countercyclical if it is domestic.[1] Nuño and Thomas (2015) and Roettger (2014) study outright default and inflation in a model with nominal debt only. The former find that real debt is welfare improving, while the latter shows that the option to default deteriorates welfare. Na et al. (2014) analyze optimal default and exchange rate policy in an open economy with downward nominal wage rigidities. They find that defaults are accompanied by large devaluations, and that default incentives are stronger in fixed-exchange rate economies, mirroring results on inflation in this paper. Different from all these papers, this paper studies the effects of ownership and denomination jointly, and highlights that their interaction is crucial in shaping macroeconomic outcomes.

Second, the paper draws on the literature of time-consistent public policy following Klein et al. (2008), Klein and Rios-Rull (2003) and Klein et al. (2005). It is most closely related to Martin (2009) and Diaz-Gimenez et al. (2008) both of which study optimal policies under lack of commitment in closed economies without outright default. Martin (2009) analyzes debt accumulation and inflation, highlighting the role that lack of commitment plays in driving both - a mechanism that is also at work in this paper. Diaz-Gimenez et al. (2008) analyze the role of debt denomination, showing that welfare effects are ambiguous and depend on

[1] This also mirrors results in Gumus (2013) who shows in a two-sector model that nominal debt (=indexed to non-tradables) reduces the countercyclicality of interest rates.

parameters and initial conditions. We build on their results and describe conditions under which nominal debt is welfare improving in a quantitative setting.

There are other papers focusing on the interaction between monetary and fiscal policy that are related to this paper but take quite different approaches. Alfaro and Kanczuk (2010) show that nominal debt is undesirable when the choice is between labor taxes and inflation, keeping debt exogenous. We show how this result is modified when we endogenize borrowing. Niemann et al. (2013) analyze inflation dynamics in a New Keynesian model with nominal debt and find that the government inflates under discretionary policy. Arellano and Heathcote (2010) study dollarization in a model of limited enforcement, finding that it increases incentives to maintain market access and thus relaxes borrowing limits. Durdu (2009) studies GDP-indexed debt which introduces state contingency in debt payments akin to nominal debt in this paper. In her setting, an intermediate degree of indexation minimizes consumption volatility, similar to the welfare results in this paper. Indexation in her framework is exogenous while we consider optimal inflation.

Finally, nominal debt and self-fulfilling sovereign debt crises are the topic of a number of recent papers, including Aguiar et al. (2013), Rocha et al. (2013), Araujo et al. (2013) and Corsetti and Dedola (2016) among others. They highlight the that there is a trade off involved in choosing to inflate - the benefits of flexibility versus the costs of distortion - which are also present in our framework of fundamentals-driven default.[2]

On the empirical side, this paper is related to the "original sin" literature beginning with Eichengreen and Hausmann (1999) that pointed out the lack of external borrowing in local currency. Our framework can contribute to an explanation for such borrowing patterns: We find that governments can sustain positive nominal external debt, suggesting that lack of commitment is not sufficient to close nominal external sovereign debt markets. A number of other papers have studied empirical government debt portfolio composition, including Reinhart and Rogoff (2011) who document the prevalence of domestic public debt, Lane and Shambaugh (2010) who study the currency composition of external debt, and a series of recent papers document increasing foreign participation in local sovereign bond markets, including Burger and Warnock (2007), Arslanalp and Tsuda (2014), Du and Schreger (2015) and Claessens et al. (2007).

[2]Several studies have explored the costs and benefits of indexed debt instruments in the context of public finance, see for instance Fischer (1975), Bohn (1990), Missale (1997) and Barro (1997), among others.

Table 1: Sovereign debt structure

	Nominal	Real
Domestic	$\delta(1-\alpha)$	$\delta\alpha$
External	$(1-\delta)(1-\kappa)$	$(1-\delta)\kappa$

2 Sovereign debt portfolios in the data

The key observation motivating this paper is that emerging market governments hold mixed debt portfolios both in terms of the residence of the investor base and the denomination of the debt. In this section we summarize these government debt portfolio patterns that will be incorporated in the theoretical model to study their implications.

We use the database compiled by Arslanalp and Tsuda (2014).[3] It contains quarterly general government debt stock time series for 24 emerging market countries from 2004Q1 to 2015Q4 by denomination and ownership. The countries are all large emerging market borrowers included in JP Morgan's emerging market bond index and its foreign currency bond index. The underlying data sources are wherever possible cross-country comparable data sources, in particular the IMF's quarterly external and public debt statistics databases (QEDS and QPDS), supplemented with national sources.

We focus on the ownership and denomination dimension of portfolios.[4] Debt is defined as domestic if the immediate holder of the bond is resident in the borrowing country. This is not necessarily related to the market of issuance as investors can purchase bonds listed on foreign exchanges, and we are only concerned with the residence of the bondholder in this paper. The important aspect of denomination is the degree to which the government has control over the real value of promised payment streams via inflation, so for the remainder of the paper, we will use "local currency" and "nominal" interchangeably, and similarly for "foreign currency", "indexed", and "real". Historically, the overlap between the sets of bonds that are held abroad and those that are real has been high - the conventional wisdom that emerging markets borrow from abroad in real terms.

We can represent the relevant government debt portfolio aspects in a matrix as in Table (1). Share δ of government debt is held domestically, of which α is denominated in real terms. Share $1-\delta$ is held abroad, of which κ is real.

The Arslanalp and Tsuda (2014) database contains sufficient information to calculate the debt shares from Table (1). It contains direct information on the share of debt that is owned externally, $1-\delta$. It also contains a measure of the external share in nominal debt,

[3] Available at http://www.imf.org/external/pubs/ft/wp/2014/Data/wp1439.zip

[4] This is separate from the jurisdiction under which bonds are issued. For an analysis of this aspect of sovereign bonds see, for example, Zettelmeyer et al. (2011) or Pitchford and Wright (2012).

Table 2: Sovereign debt portfolios - Summary statistics

	δ	κ	α
Mean	0.67	0.61	0.18
Median	0.65	0.63	0.07
SD	0.19	0.26	0.22

$\frac{(1-\delta)(1-\kappa)}{\delta(1-\alpha)+(1-\delta)(1-\kappa)}$, and of the nominal share in total debt, $\delta(1-\alpha)+(1-\delta)(1-\kappa)$. Using these three pieces of information, we can back out δ, α, κ, the portfolio shares of interest.[5] Since there is less variation within countries over time in the portfolio breakdown than across countries, we take time-averages of portfolio shares for each country.

Table (2) shows summary statistics of the resulting portfolio shares.[6] It illustrates three main empirical regularities: Domestic debt shares are large, the majority of domestic debt is nominal, and relatively little of external debt is nominal. In other words there is some correlation between domestic debt being nominal, but it is less than perfect. Importantly, it shows that only a relatively small share of around 20% of debt is real and external ($\kappa(1-\delta)$) which is the most common assumption when studying sovereign debt problems in emerging market countries.

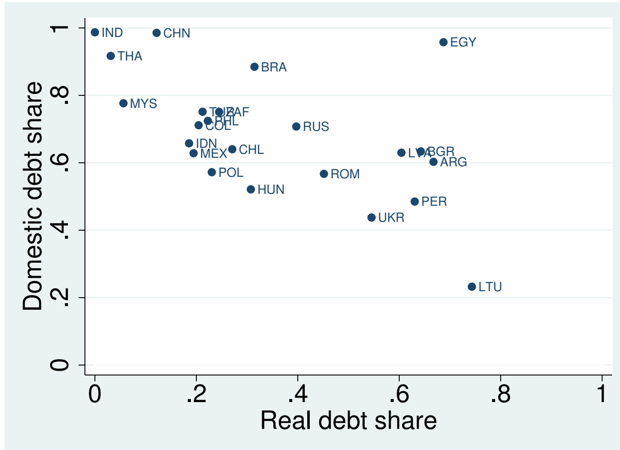
Figure (1) plots the cross-section of portfolios in the domestic-real share plane similar to Table (1). The real share is computed as the average of domestic and external real debt, weighted by the domestic and external shares.[7] The Figure shows that in terms of the quadrants from Table (1), there are relatively few countries in the top right or bottom left quadrants. The majority of sovereigns borrows largely from foreigners in real terms, or from their own citizens in nominal terms. Asia tends to occupy the top left domestic/nominal part of the graph, while Argentina is close to the bottom right external/ real corner. Overall, there is substantial heterogeneity in portfolio composition across countries.

[5]See the appendix for details.

[6]See the appendix for the underlying time series for each country.

[7]That is $\delta\alpha+(1-\delta)\kappa$

Figure 1: Portfolio composition in the cross-section



3 Model

We now develop a theory of sovereign borrowing, inflation and default to characterize and quantify the role of debt denomination and ownership in shaping equilibrium outcomes. The model is a small open economy in discrete and infinite time with households, the government and international investors. Households work and save in money and government bonds to smooth consumption over time. They are subject to a cash in advance constraint on consumption and shocks to labor productivity. The government raises taxes, prints money and issues debt to maximize household welfare. It borrows in bonds that differ along the two key dimensions of interest: Denomination (nominal or real) and ownership (external or domestic). It lacks commitment to its policies, so that it has an incentive to borrow, inflate and default outright.

3.1 Debt structure

We assume the following debt structure. The government issues one-period claims that can be bought by foreign or domestic bondholders. For every unit bond issuance, fraction δ is bought by domestic residents and fraction $1-\delta$ is bought by international investors. Fraction α of domestic holdings and fraction κ of external holdings are indexed to the domestic price level, while the remaining fractions are nominal. The government takes this structure as given, and we only need to consider optimal total debt issuance b when solving its optimal policy problem.

For concreteness, assume that the government issues b bonds today. If the entire issuance is bought by international investors ($\delta = 0$) and is indexed to the price level ($\kappa = 1$), then b represents a promise to repay b units of consumption tomorrow to the foreign investors, just like in a standard sovereign default model with external debt only. If the bonds are nominal ($\kappa = 0$), issuing b represents a claim to b pesos tomorrow. This claim is worth b/P_t units of consumption today, but only b/P_{t+1} when repaid tomorrow, where P_t and P_{t+1} are the domestic price levels in the two periods. When inflation $\pi_{t+1} = P_{t+1}/P_t - 1$ is positive, the real value of this nominal debt will be eroded. In general, b is a promise to pay $(\kappa+\alpha)b$ units of consumption pus $((1-\kappa)+(1-\alpha))b$ pesos, split between foreigners and locals according to δ.

We restrict attention to exogenous, fixed portfolio structures and analyze their effects since this has not been studied in the literature and they are not well understood.[8] In the data, the cross-country heterogeneity in debt portfolio structures is larger than the within country variation, making it not implausible to restrict attention to exogenous, average portfolios in a given country. Moreover, debt management offices often publish and pursue longer term strategies with regard to denomination and market of issuance targets for their debt, that is separate from day to day bond issuances. We also abstract from the role of exchange rates as a separate policy tool. Since the law of one price holds in this economy, so the nominal exchange rate is unity and there is no distinction between indexed and foreign currency bonds. We focus on inflation instead since Mexico and many of the countries in the sample have floating exchange rates.[9]

3.2 Households

The representative agent maximizes the discounted expected lifetime utility from cash good consumption, credit good consumption and labor:

$$U = E_0 \sum_{t=0}^{\infty} \beta^t u(c_{1t}, c_{2t}, n_t) \tag{1}$$

where β is the subjective discount factor and we assume

$$u(c_1, c_2, n) = \gamma \frac{c_1^{1-\sigma}}{1-\sigma} + \log c_2 - \phi \frac{n^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}}$$

Preferences are separable and logarithmic in credit good consumption. The Frisch elasticity of labor supply is assumed to be constant and equal to ν. The curvature σ and weight on cash good consumption γ will determine the extent to which money and inflation are

[8]See Engel and Park (2016) for an example of endogenous portfolio choice with respect to denomination, not ownership.

[9]See for example Na et al. (2014) for a model with foreign currency bonds and exchange rates as the policy tool.

important in the economy.

Households buy cash and credit goods, as well as new money balances and, if the government is in good credit standing, bonds to carry into the next period. They finance their expenditures with labor income net of taxes, existing money balances and existing savings in government bonds. Their budget constraint is thus:

$$c_{1t} + c_{2t} + \frac{\bar{m}_{t+1}}{\bar{p}_t} + (1 - d_t)\left(\frac{1-\alpha}{\bar{p}_t} + \alpha\right) q_t \delta b_{t+1} = (1-\tau_t) w_t n_t + \frac{\bar{m}_t}{\bar{p}_t} + (1-d_t)\left(\frac{1-\alpha}{\bar{p}_t} + \alpha\right)\delta b_t$$

where d_t is a binary indicator equal to 1 if the government chooses to default and 0 otherwise.[10] The within-period timing of events follows Svensson (1985): We assume that cash goods can only be purchased with existing money balances so that households are subject to cash-in-advance (CIA) constraint

$$\bar{p}_t c_{1t} \leq \bar{m}_t \tag{2}$$

This makes expected inflation costly if it constrains and thus distorts optimal cash good consumption, and makes unexpected inflation costly because the real value of existing money balances is lower than households budgeted for at the end of the previous period. Credit good purchases $\bar{p}_t c_{2t}$ can be financed with all income.

In order to make this problem stationary, we divide the nominal variables $\bar{p}_t$ and $\bar{m}_t$ as well as the nominal portion of bonds, $(1-\alpha)\delta$ by the aggregate money supply, $\bar{M}_t$, following Cooley and Hansen (1991).[11] Defining the money growth rate between tomorrow and today as $\mu_t \equiv \frac{\bar{M}_{t+1}}{\bar{M}_t} - 1$, we can then write the normalized household budget constraint as

$$c_{1t} + c_{2t} + \frac{1+\mu_t}{p_t} m_{t+1} + \left(\frac{(1-\alpha)(1+\mu_t)}{p_t} + \alpha\right)(1-d_t) q_t \delta b_{t+1} = (1-\tau_t) w_t n_t + \frac{m_t}{p_t} + \left(\frac{1-\alpha}{p_t} + \alpha\right)(1-d_t)\delta b_t \tag{3}$$

[10] Here d is equal to 1 both of the government has chosen to default in the current period, and if he has chosen to default in a prior period and has not yet regained market access, so his credit standing is bad. We will distinguish between the default decision and the credit standing in the recursive formulation of the government problem below.

[11] We should write bond service as $\frac{1-\alpha}{\bar{p}_t}\delta \bar{b}_t + \alpha \delta b_t$ (and analogously bond revenue), but after normalizing $\frac{\bar{b}}{\bar{M}} = b$ because of market clearing so we go straight to b for notational simplicity.

3.2.1 Solution to the household problem

We characterize the solution to the household problem conditional on government policies. The government will solve its optimal policy problem subject to these competitive equilibrium conditions of the household. A solution to the household problem are sequences $\{c_{1t}, c_{2t}, n_t, m_{t+1}, b_{t+1}\}_{t=0}^{\infty}$ that maximize (1) subject to (2) and (3), taking as given price sequences $\{\mu_t, q_t, w_t, \tau_t, p_t\}_{t=0}^{\infty}$, a sequence of credit standings $\{d_t\}_{t=0}^{\infty}$ and initial money and bond holdings m_0 and b_0. The associated first order conditions that have to hold in every period are given by:

$$\frac{-u_{3t}}{u_{2t}} = (1-\tau_t)w_t \tag{4}$$

$$u_{2t}\frac{1+\mu_t}{p_t} = \beta E_t\left[\frac{u_{1t+1}}{p_{t+1}}\right] \tag{5}$$

$$u_{2t}\left(\frac{(1-\alpha)(1+\mu_t)}{p_t}+\alpha\right)q_t = \beta E_t\left[(1-d_{t+1})u_{2t+1}\left(\frac{1-\alpha}{p_{t+1}}+\alpha\right)\right] \tag{6}$$

$$u_{1t}-u_{2t} \geq 0 \tag{7}$$

where expectations are taken over the productivity shock tomorrow, conditional on the realization today and u_i denotes the partial derivative of the utility functions with respect to the i^{th} argument.

The first equation is the standard intratemporal first order condition that characterizes the optimal trade-off between credit good consumption and leisure. The last condition arises from the CIA constraint. If the constraint is not binding, the condition holds with equality and consumption is not distorted - marginal utilities are equalized. Equation (6) is an Euler condition for the marginal utility of credit good consumption in periods t and $t+1$. We can rearrange it as a standard asset pricing equation to see that the bond price compensates domestic bondholders for consumption, default and inflation risk:

$$q_t = \beta E_t\left[\underbrace{\frac{u_{2t+1}}{u_{2t}}}_{\substack{\text{Consumption}\\\text{risk}}}\underbrace{(1-d_{t+1})}_{\text{Default risk}}\underbrace{\frac{\frac{1-\alpha}{p_{t+1}}+\alpha}{\frac{(1-\alpha)(1+\mu_t)}{p_t}+\alpha}}_{\text{Inflation risk}}\right] \tag{8}$$

Everything else equal, a high stochastic discount factor increases household savings demand and thus the bond price. A high probability of default in the next period lowers the price and raises the interest rates that households demand in order to hold government bonds.

When all bonds are nominal ($\alpha = 0$), expression (8) reduces to

$$q_t = \beta E_t \left[\frac{u_{2t+1}}{u_{2t}} (1 - d_{t+1}) \frac{1}{1 + \pi_{t+1}} \right]$$

where

$$1 + \pi_{t+1} \equiv \frac{p_{t+1}(1 + \mu_t)}{p_t} \tag{9}$$

is consumer price inflation.[12] This states that households need to be compensated to hold bonds whose real payout is expected to be eroded through price inflation. In the solution to the optimal policy problem, the government will internalize that higher inflation and default risk reduce bond prices according to this pricing equation, which will limit its borrowing in equilibrium.

First order condition (5), finally, can shed light on how the cash in advance constraint distorts the economy. Rearranging we can write it as

$$1 = \beta E_t \left[\frac{u_{1t+1}}{u_{2t}} \frac{1}{1 + \pi_{t+1}} \right]$$

This equation shows that in an undistorted steady state, the Friedman rule holds. In a steady state, when the cash in advance constraint does not bind, we have $u_1 = u_2$ from equation (7). In that case inflation is equal to β, the inverse of the real risk free rate, and the nominal interest rate is zero. (Expected) inflation is positive, therefore, either if the cash in advance constraint binds in steady state ($u_1 > u_2$), or outside a steady state if marginal utility of cash consumption is expected to be sufficiently high ($u_{1t+1} > u_{2t}$): Households are willing to pay a positive nominal interest rate to hold money balances if they expect to be cash constrained.

3.3 External investors

External debt is assumed to be bought by international investors in a competitive market. Their opportunity cost is the international real risk free rate r, and they choose government bond purchases, b_{t+1}, in order to maximize expected profits. International investors, unlike domestic residents, are risk neutral so their stochastic discount factor does not affect the pricing of the external debt. This assumption seems plausible for the frequently large, diversified, and institutional external investors that buy emerging market government debt.

[12] Recall that p is the nominal price level scaled by the aggregate money stock, that is $p \equiv \frac{\bar{p}}{\bar{M}}$, and that $1 + \mu_t \equiv \frac{\bar{M}_{t+1}}{\bar{M}_t}$. Then $1 + \pi_t \equiv \frac{\bar{p}_t}{\bar{p}_{t-1}} = \frac{p_t}{p_{t-1}} \frac{M_t}{M_{t-1}} = \frac{p_t(1+\mu_{t-1})}{p_{t-1}}$

Their static maximization problem in each period t is

$$\max_{b_{t+1}} \Pi_t = q_{et}\left(\frac{(1-\kappa)(1+\mu_t)}{p_t}+\kappa\right)b_{t+1} - \frac{1}{1+r}E_t\left[(1-d_{t+1})\left(\frac{1-\kappa}{p_{t+1}}+\kappa\right)\right]b_{t+1}$$

which pins downs the price of external debt as

$$q_{et} = \frac{1}{1+r}E_t\left[(1-d_{t+1})\frac{\frac{1-\kappa}{p_{t+1}}+\kappa}{\frac{(1-\kappa)(1+\mu_t)}{p_t}+\kappa}\right] \tag{10}$$

This expression is similar to the pricing of domestic debt: Bondholders are compensated for repayment risk and, to the extent that $\kappa < 1$, inflation risk. In the case $\kappa = 1$, the expression reduces to the standard bond pricing formula for external, real sovereign debt with repayment risk only.

3.4 Government, production, and market clearing

The government finances exogenous expenditures g using labor income taxes, seigniorage and net bond revenues, so we can write its budget constraint as follows:

$$\begin{aligned} g =& \tau_t w_t n_t + \frac{(1+\mu_t)}{p_t} - \frac{1}{p_t} \\ &+ (1-d_t)\left\{\left[\delta q_t\left(\frac{(1-\alpha)(1+\mu_t)}{p_t}+\alpha\right) + (1-\delta)q_{et}\left(\frac{(1-\kappa)(1+\mu_t)}{p_t}+\kappa\right)\right]B_{t+1}\right. \\ &\left. - \left[\delta\left(\frac{1-\alpha}{p_t}+\alpha\right) + (1-\delta)\left(\frac{1-\kappa}{p_t}+\kappa\right)\right]B_t\right\} \end{aligned} \tag{11}$$

where $\tau_t w_t n_t$ is labor tax revenue, $\frac{\mu_t}{p_t}$ is real seigniorage revenue, the terms on the second line are revenue from new bond issuances, and the last line captures debt repayments. The second and third lines are conditional on good credit standing, $d_t = 0$. We include labor income taxes as an empirically important source of revenue for most governments, and in order to allow the model to replicate a quantitatively realistic revenue split between bonds, seigniorage and taxes. Omitting tax revenue would exaggerate the importance of seigniorage relative to what we see in the data.

The production side of the economy is simple with output being produced using a linear technology,

$$y_t = z_t n_t \tag{12}$$

where z_t is an exogenous productivity shock, y_t is output and n_t labor supply. Labor will be paid its marginal product, so that in equilibrium $w_t = z_t$.

In equilibrium, money balances held by domestic households must equal money supplied by the government: $\bar{m}_t = \bar{M}_t, \forall t$, and thus normalized money supply $m_t = M_t = 1, \forall t$. Similarly for bond markets, bonds held must equal bonds supplied, $B_t = \delta b_t + (1-\delta) b_t = b_t, \forall t$. The cash in advance constraint will hold with equality in equilibrium (but not necessarily bind) since households at the margin prefer to consume rather than carry over money balances that at best have a zero return, so

$$c_{1t} = \frac{1}{p_t} \tag{13}$$

Finally the resource constraint of the small open economy, derived by combining the household and government budget constraints after imposing money and bond market clearing, will have to hold at all points in time:

$$c_{1t} + c_{2t} + g + x_t \quad = \quad y_t \tag{14}$$

where x_t is equal to net external savings:

$$x_t \quad \equiv \quad (1-\delta)\left[\left(\frac{(1-\kappa)}{p_t} + \kappa\right) b_t - \left(\frac{(1-\kappa)(1+\mu_t)}{p_t} + \kappa\right) q_{et} b_{t+1}\right] \tag{15}$$

The benefits of default are twofold in the model, as is clear from these constraints: One the one hand, it has tax smoothing benefits via the government budget constraint. A reduced debt burden allows the government to reduce distortionary labor taxes and seigniorage. On the other, to the extent that debt is external, default wipes out negative net external savings and thus directly increases the resources available for consumption via the resource constraint. These two motives are typically what drives default in the literature on either domestic or external default, and here they are combined in the presence of both types of debt. Benefits of default are higher in the case of external debt since it facilitates both an increase in resources available for consumption and a possible reduction in distortionary taxes.

4 Optimal policy problem

Equilibrium concept The government in the economy is benevolent but lacks commitment to its policies. This lack of commitment introduces incentives to borrow since debt issuance today is non-distortionary, and incentives to erode the real value of outstanding debt through default or inflation. We analyze the time-consistent Markov perfect equilibrium of the economy, in line with the literature (e.g. Klein et al. 2008 and Arellano 2008). In such an equilibrium, the government's optimal policies are functions of the current state of the economy only, the productivity shock z and its debt position b. When solving its problem, the government takes as given the policy functions of future governments, and so it internalizes how its choice of borrowing affects the future state of the economy and future policies. In particular, it sees that borrowing today raises default and inflation incentives in the future.

Sovereign default We assume that default entails costs typically assumed in the literature: A drop in productivity and temporary market exclusion. The former captures in a simple way disruptions to the real side of the economy, the latter periods of no debt issuance observed in the data following debt crises.

We abstract from debt renegotiations for simplicity and assume that debt is written down completely. Allowing for non-zero debt recovery rates would, everything else equal, reduce default incentives uniformly across bond types.[13]

The government is assumed to default simultaneously on domestic residents and foreigners and cannot discriminate. This assumption has support in the data. Empirically, sovereign bonds are frequently structured such that default in one triggers default in another via cross-default and acceleration clauses. Choi et al. (2011) for example find that 85% of Brady issuers included cross-default and acceleration clauses in their international bond issues between 1982 and 2000, and 63% of other sovereigns (excluding very safe borrowers like the US and Germany). Trebesch et al. (2012) document that collective action clauses (CACs), including cross-default and acceleration clauses, have become well-established market practice for bonds issued under international law. They show that Mexico in particular, which we will calibrate the model to later, has since 2003 issued more than 90% of its sovereign bonds under New York law which typically includes CACs.[14]

Implementability constraints The government when solving its problem takes into account that the solution must be consistent with the competitive equilibrium conditions from the household problem. It is therefore helpful to rewrite the government budget constraint

[13] In particular, there is no systematic evidence of discrimination against groups of bondholders in renegotiations. Trebesch et al. (2012) find in a comprehensive review of sovereign defaults globally that foreign bondholders have not received systematically unfavorable terms relative to domestic bondholders.

[14] Other support for the prevalence of simultaneous default on domestic and foreign bondholders includes Erce and Diaz-Cassou (2010) who show that seven out of eleven external restructurings in a sample of recent debt crises were preceded or followed by domestic debt restructurings.

(11) in terms of allocations only using these competitive equilibrium conditions. We can substitute out the money growth rate, the tax rate and the domestic bond price using expressions (4) through (6), and substitute out the price level using (13) to get:[15]

$$u_2(c_2)\left((1-d)\left(c_1(1-\alpha)+\alpha\right)\delta b - c_2\right) - u_3(n)n = \\ (1-d)\left(\zeta_1^1(z,b') + \zeta_2^1(z,b')\delta b'\right) + d\zeta_1^0(z) \tag{16}$$

$\zeta_2^1(z,b')\delta b'$ is revenue from selling domestic bonds, and $\zeta_1^1(z,b')$ and $\zeta_1^0(z)$ are revenue from printing money in repayment and default, respectively:

$$\zeta_1^1(z,b') \equiv \beta E_{z'|z}\left[(1-\boldsymbol{d}(z',b'))u_1\left(\boldsymbol{c_1^1}(z',b')\right)\boldsymbol{c_1^1}(z',b')\right. \tag{17}$$

$$\left. +\boldsymbol{d}(z',b')\boldsymbol{c_1^0}(z')u_1\left(\boldsymbol{c_1^0}(z')\right)\right] \tag{18}$$

$$\zeta_1^0(z) \equiv \beta\left(\eta\zeta_1^1(z',0) + (1-\eta)E_{z'|z}\left[\boldsymbol{c_1^0}(z')u_1\left(\boldsymbol{c_1^0}(z')\right)\right]\right) \tag{19}$$

$$\zeta_2^1(z,b') \equiv \beta E_{z'|z}\left[(1-\boldsymbol{d}(z',b'))u_2\left(\boldsymbol{c_2^1}(z',b')\right)\left(\boldsymbol{c_1^1}(z',b')(1-\alpha)+\alpha\right)\right] \tag{20}$$

Since utility is separable, we include with a slight abuse of notation only the dependence on the respective argument in the $u_i(\cdot)$ expressions.

The expression for money revenues in default, (19), captures our assumption that all outstanding debt is written off and the government cannot issue new bonds in default, but has a η chance each period of re-entering capital markets with zero debt. The assumptions of a complete debt writedown implies that the functions in bad credit standing do not depend on debt or borrowing.

We can rewrite the resource constraint (14) in a similar way as the government budget constraint. Substituting out the external bond price (10) (and again using (13)), the resource constraint becomes

$$c_1 + c_2 + g + (1-d)(1-\delta)\left[\left(c_1(1-\kappa)+\kappa\right)b - \zeta_3^1(z,b')b'\right] = zn \tag{21}$$

$$\zeta_3^1(z,b') \equiv \frac{1}{1+r}E_{z'|z}\left[(1-\boldsymbol{d}(z',b'))\left(\boldsymbol{c_1^1}(z',b')(1-\kappa)+\kappa\right)\right] \tag{22}$$

with $\zeta_3^1(z,b')(1-\delta)b'$ the revenue from issuing external bonds. Notice that written in this way, all revenues from current period bond issuances ζ_1, ζ_2 and ζ_3 are independent of outstanding debt b and solely a function of borrowing b'. They describe how the government generates revenue from selling new bonds, and how this revenue is affected by future policies. We will return to these expressions in the next section after defining the government problem and equilibrium.

[15] See the appendix for the derivation.

Problem 1 (Government Problem). If the government is in good credit standing, it has the choice of whether to remain current or default:

$$V(z,b) = \max_{d \in \{0,1\}} (1-d)V^1(z,b) + dV^0(z) \tag{23}$$

In case of repayment, it chooses allocations b', c_1, c_2 and n subject to the implementability constraint, resource constraint and the household FOC for the cash in advance constraint:

$$\begin{gathered} V^1(z,b) = \max_{b',c_1,c_2,n} u(c_1,c_2,n) + \beta E_{z'|z}[V(z',b')] \\ \text{subject to (7),(16) and (21)} \end{gathered} \tag{24}$$

In case of default, it chooses c_1, c_2 and n subject to the same constraints, and re-enters capital markets with no debt in the next period with probability η:

$$\begin{gathered} V^0(z) = \max_{c_1,c_2,n} u(c_1,c_2,n) + \beta E_{z'|z}[\eta V(z',0) + (1-\eta)V^0(z')] \\ \text{subject to (7),(16) and (21)} \end{gathered} \tag{25}$$

Definition 1 (Markov Perfect Equilibrium). *A Markov perfect equilibrium of the economy are value function $V(z,b)$ with associated policy function $\boldsymbol{d}(b,z)$, value function in repayment $V^1(z,b)$ with associated policy functions $\boldsymbol{h}(z,b)$, $\boldsymbol{c_1^1}(z,b)$, $\boldsymbol{c_2^1}(b,z)$, $\boldsymbol{n^1}(b,z)$, and value function in default $V^0(z)$ with associated policy functions $\boldsymbol{c_1^o}(z)$, $\boldsymbol{c_2^0}(z)$, and $\boldsymbol{n^0}(z)$ that solve (23), (24) and (25), with $b' = \boldsymbol{h}(z,b)$, $d = \boldsymbol{d}(z,b)$, and for (24) in good credit standing $c_1 = \boldsymbol{c_1^1}(z,b)$, $c_2 = \boldsymbol{c_2^1}(z,b,)$, $n = \boldsymbol{n^1}(z,b)$, while for (25) in bad credit standing $c_1 = \boldsymbol{c_1^0}(z)$, $c_2 = \boldsymbol{c_2^0}(z)$, $n = \boldsymbol{n^0}(z)$.*

Equilibrium prices can be read off the competitive equilibrium conditions (4) through (6), (10) and (13).

4.1 Government incentives

Euler equation The solution to the optimal policy problem is characterized by an intertemporal Euler equation that we can use to understand the trade-offs that the government faces. The problem is in general non differentiable because of the presence of default, but conditional on repaying tomorrow we can write down the first order condition for an interior borrowing choice.

Denote by λ_1 and λ_2 the Lagrange multipliers on the implementability constraint (16) and the resource constraint (21) respectively, let $\mathcal{Z}(b)$ be the set of z for which the government does not default given b, and let $\hat{\zeta}_i^1(z,b)$ be defined such that $\zeta_i^1(z,b') = E_{z'|z}[\hat{\zeta}_i^1(z',b')], i = 1,2,3$. Then the government's Euler equation is given by:[16]

$$\lambda_1 \left(\delta \left(\zeta_2^1(z,b') + b' \frac{\partial \zeta_2^1(z,b')}{\partial b'} \right) + \frac{\partial \zeta_1^1(z,b')}{\partial b'} \right) + \lambda_2 (1-\delta) \left(\zeta_3^1(z,b') + b' \frac{\partial \zeta_3^1(z,b')}{\partial b'} \right) = \beta E_{z'|z} \left[\left(\lambda_1' \delta \hat{\zeta}_2^1(z',b') + \lambda_2'(1-\delta) \hat{\zeta}_3^1(z',b') \right) | z' \in \mathcal{Z}(b') \right] \quad (26)$$

This equation describes how the government trades off costs and benefits of additional borrowing. On the right hand side, we have the marginal costs of an additional unit of bonds issued. Borrowing more today implies that debt is high tomorrow, and so both the government budget constraint and the resource constraint are tighter as the government faces higher debt service to both households and foreigners. This is captured by their respective future Lagrange multipliers λ_1' and λ_2', weighted by the size of the repayment that has to be made for each unit of debt. These costs are balanced against the marginal benefits of borrowing on the left hand side of the equation. Issuing an additional unit of bonds yields direct revenue of $\delta\zeta_2^1$ and $(1-\delta)\zeta_3^1$, but it also affects future default, inflation and consumption risk and thus changes the revenue that government can generate from b' today, as shown by the presence of the derivatives of ζ_1^1, ζ_2^1 and ζ_3^1.

The sign of the derivatives in the Euler equation shapes the government's incentives to borrow. If high bond issuance today raises default and inflation risk, and thus lowers bond prices, for example, $\frac{\partial \zeta_2^1(z,b')}{\partial b'}$ and $\frac{\partial \zeta_3^1(z,b')}{\partial b'}$ are negative, and provide a disincentive for the government to borrow more. Borrowing affects household money demand and thus government seigniorage revenue. If money demand tends to increase with debt, $\frac{\partial \zeta_1^1(z,b')}{\partial b'} > 0$, the government recognizes this as an additional benefit to borrowing more.

We will next use a numerical example from the calibrated model to study these revenue functions, and their slopes, in more detail.

Revenue sources The government's revenue functions are shown graphically in Figure (2). We plot the expressions ζ_1^1, ζ_2^1 and ζ_3^1 from equations (17), (20) and (22) along with the default probability as a function of borrowing b' for a range of productivity levels z.

The marginal revenue of external bond revenue, ζ_3^1, shown in the bottom right, is the analogue to the bond price in a standard sovereign default model except that the price includes inflation compensation if external debt is nominal, and thus exhibits a familiar shape: At low levels of borrowing and for relatively high productivity, when default risk plays no role, ζ_3^1 is below the international risk free rate only to the extent that external

[16] See the appendix for the derivation.

debt is nominal and subject to inflation risk. The function is decreasing since prices rise (as per equation (22)), but this effect is minor and visually barely detectable. For sufficiently high borrowing or low productivity, default risk dominates the shape of the revenue function.

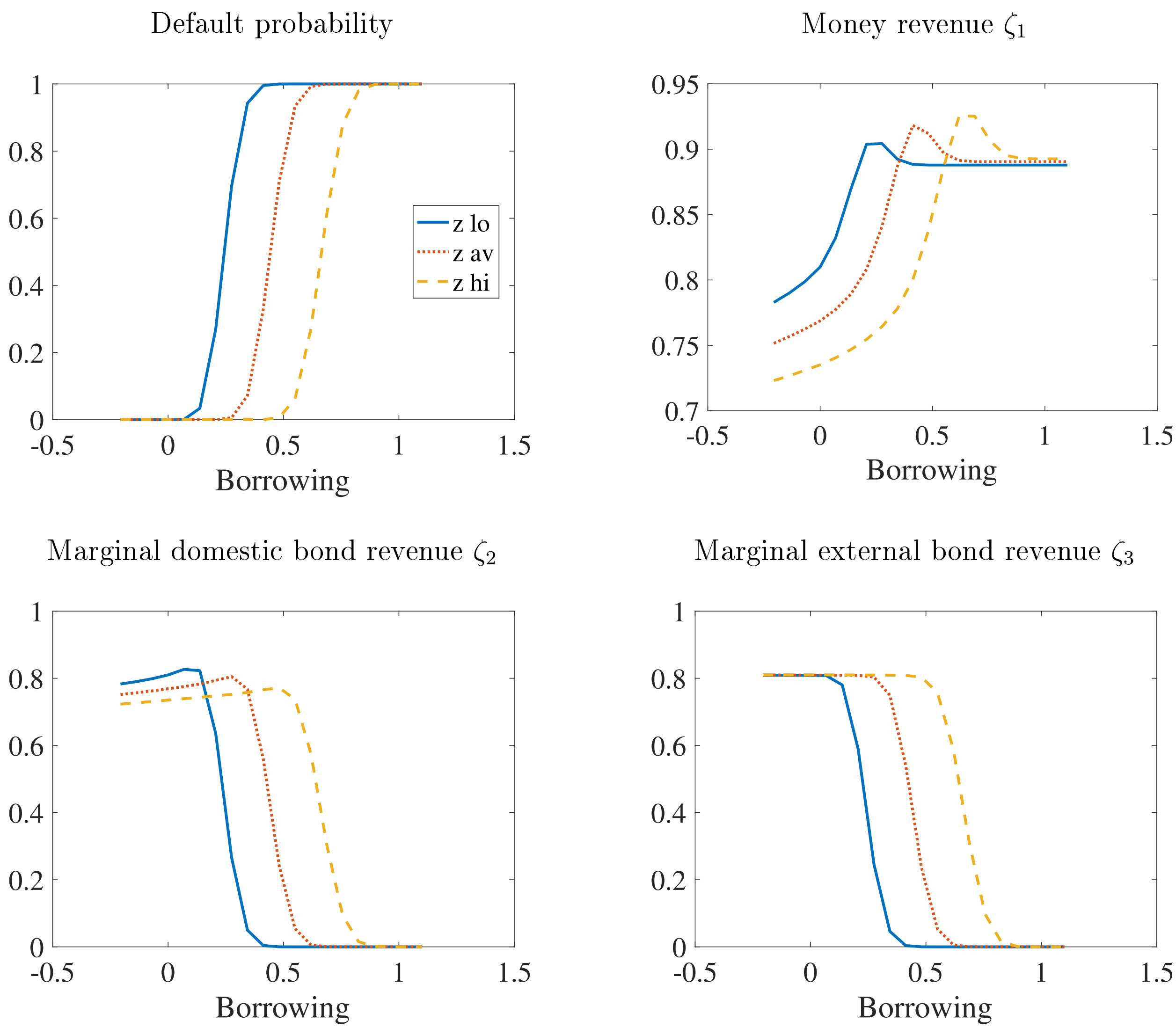
The marginal revenue of domestic bonds, ζ_2^1, shown in the bottom left panel is broadly similar to the revenue from external bonds ζ_3^1, but with some differences: At low levels of borrowing when default risk is zero, additional borrowing raises the marginal revenue from issuing domestic bonds. This is because high levels of borrowing for households signal lower consumption in future periods (high $u_2(\boldsymbol{c_2^1}(z,b))$ in equation (20)) as meeting debt service obligations requires higher net external savings and distortionary taxes. As a result, households increase their demand for government bonds to self insure and smooth out the anticipated consumption drop. This benefits the government who now obtains higher marginal revenues from borrowing from domestic households. In addition, the marginal domestic bond revenue is decreasing in productivity at borrowing levels where default risk is negligible. This occurs for similar reasons: Expected marginal utility is relatively high in bad times, so everything else equal households prefer to have savings at their disposal that they can run down tomorrow.

Revenue from printing money finally, ζ_1^1, exhibits a Laffer curve shape that mirrors total bond revenues (rather than the marginal revenues depicted): It is increasing in borrowing as long as default risk is sufficiently low, but decreasing beyond a certain point. Equation (17) shows that borrowing affects it through similar channels as in the case of bond revenue. On the one hand, more borrowing means higher prices (low $\boldsymbol{c_1^1}(z,b)$) and thus a lower real value of money balances tomorrow. This reduces households' willingness to hold money today. On the other hand, households anticipate a high marginal utility of cash consumption (high $u_1(\boldsymbol{c_1^1}(z,b))$) when borrowing is high, which increases their money demand. They expect to be cash constrained and are willing to hold money to guard against that. This second factor dominates here, and borrowing increases money revenues for the government, as long as it is not in default. ζ_1 remains positive even in default since even in default the government prints money and generates seigniorage.

Inflation and borrowing What can we say about the effect of borrowing on inflation risk? Borrowing turns out to increase inflation risk, but only up to a point, mirroring the revenue from money creation. To see this, write inflation between today and tomorrow as follows

$$1+\pi(z,z',b') = \frac{c_1(1+\mu(z,b'))}{\boldsymbol{c_1^1}(z',b')} = \frac{1}{u_2}\frac{\zeta_1^1(z,b')}{\boldsymbol{c_1^1}(z',b')} \tag{27}$$

Figure 2: Government revenues as a function of borrowing



where we have used equation (5) and equation (9). In equilibrium c_1 and u_2 will depend on z and b, but we suppress this dependence in the expression and focus only on the effect of borrowing, b'.

Cash consumption $\boldsymbol{c_1}(z', b')$ in the denominator is decreasing in b', which tends to increase inflation. $\zeta_1^1(z, b')$ is increasing in b' up to a point when default risk kicks in, as we have seen. So as long as default risk plays no large role, borrowing drives up inflation. However, increasing borrowing beyond a certain point will raise default risk and thus may, depending on the relative changes in the ratio $\frac{\zeta_1^1(z,b')}{\boldsymbol{c_1^1}(z',b')}$, actually lower inflation.

Intuitively, in default the government faces weaker inflation incentives since it has no debt to service, and so when default risk becomes sufficiently high, it becomes increasingly likely that the government will not need to inflate tomorrow because it defaults instead.

Summarizing, we have shown that the presence of money and domestic debt gives the government at the margin additional reasons to accumulate debt, while default risk continues to provide disincentives to borrow. Whether stronger incentives to borrow in the presence of nominal and domestic debt translate into higher inflation and default rates depends on how much debt the government chooses to accumulate in equilibrium. This is what we turn to next - the calibration and quantitative model results.

5 Quantitative analysis of debt portfolios

5.1 Calibration

The model is calibrated to Mexico, both for reasons of data availability and because it has a relatively mixed government debt portfolio along the dimensions of interest to this paper. We pick the parameters governing the debt portfolio to replicate its public debt composition.

Table 3: Parameters I

Parameter			Description/ target
Domestic debt share	δ	0.65	Mexico debt portfolio
Real external debt share	κ	0.78	Mexico debt portfolio
Real domestic debt share	α	0.00	Mexico debt portfolio
Int'l real risk free rate	r	0.0034	90-day US Treasury bill yield 1.37% annually
Subjective discount factor	β	0.98	Mexico real risk free rate 8.4% annually
Re-entry probability	η	0.25	Average exclusion duration 1 year
Frisch elasticity	ν	2.00	Standard macro estimate
Government spending	g	0.0363	Average government spending 11% of GDP
Productivity persistence	ρ	0.95	Neumeyer and Perri (2005)

Table 4: Parameters II

Parameter			Description/ target	Data	Model
Productivity volatility	σ_ϵ	0.015	Mexico GDP volatility	0.018	0.018
Labor weight in utility	ϕ	10.00	Fraction of time working	0.33	0.33
Cash good weight in utility	γ	0.025	Cash/credit good ratio	0.82	0.82
Curvature cash good	σ	2.80	Inflation rate	0.042	0.040
Default cost threshold	χ	-0.10	Default rate	0.020	0.018

Between 2004Q1 and 2015Q4, the years for which data is available, its debt was on average 65% domestically owned, 78% of which was in real terms, while all of the domestic debt was nominal. We thus set $\delta = 0.65, \kappa = 0.78$ and $\alpha = 0$.

We assume that log productivity follows a stationary AR(1) process

$$\log z_t = \rho \log z_{t-1} + \epsilon_t, \epsilon \sim N(0, \sigma_\epsilon^2)$$

In choosing the persistence and volatility of the productivity process, we follow Neumeyer and Perri (2005) and set $\rho = 0.95$ while calibrating σ_ϵ to match the volatility of quarterly Mexican HP-filtered log real GDP from 1997Q1, the end of its last debt crisis, through 2015Q4.

For the default cost, we follow Arellano (2008) and assume that in default, productivity is given by

$$z_{def} = \max\{z, \chi\}$$

and set the parameter χ to match an annual default rate of 1.6%.[17] Mexico did not default since 2004, when our debt portfolio data starts, and defaulted three times against the International Bank of Reconstruction and Development (IBRD) in the postwar period according to Asonuma and Trebesch (2016) which implies an annual default rate of around 4%. We choose a rate half that, 2%, to capture the fact that there is default risk but that we have observed no defaults in sample. We do not go back further than the postwar period to calculate default rates to avoid including times with very different macroeconomic conditions and most likely public debt portfolios.

The probability of market re-access following a default is set to $\eta = 0.25$ implying an average exclusion period of 1 year. This is in line with estimates by Asonuma and Trebesch (2016) according to which Mexico renegotiated its debt within around 1 year of defaulting.

[17]It is quantitatively important here, as well as in other studies, that the outright default costs are asymmetric and higher in times of high productivity. The asymmetry implies that the sovereign accumulates significant amounts of debt in good times, when he has strong disincentives to default. See the appendix for a plot of the resulting effective productivity.

The international risk free rate is set to $r = 0.0034$, which corresponds to an average annualized interest rate of 1.37% on 90-day US T-bills between 1997Q1 and 2015Q4. We choose the disutility of labor ϕ to match a fraction of time spent working of 33%, which then determines $g = 0.0363$ to give average government spending to GDP of 11% between 1997Q1 and 2015Q4. The Frisch elasticity of labor supply is set to $\nu = 2$, within the standard range of macro estimates for the elasticity of aggregate hours,[18] and we set $\beta = 0.98$ for an annual domestic real risk free rate of 8.4%, equal to the long run return to capital in Mexico since 1950.

The parameter governing the elasticity of cash good consumption, σ, is chosen to match average consumer price inflation of 4.2% annually between 2004Q1 and 2015Q4, where inflation is computed using quarterly CPI data from the IMF.

The weight of cash goods in utility γ is set to match the cash to credit good ratio observed in the data in order to capture the extent to which households are exposed to monetary distortions. Let ϕ be the long term share of cash consumption in production net of national savings, $c_1 = \phi(y - g - x)$, so the targeted cash credit ratio is given by $\frac{c_1}{c_2} = \frac{\phi}{1-\phi}$. If $c_1 = M/p$, as is the case in the model in equilibrium, we can express ϕ as a function of observables: $\phi = \frac{M}{P(y-g-x)}$. Using quarterly data on nominal GDP, government spending, net exports and the M1 money stock, this implies an average cash/credit good ratio of 0.82 in Mexico between 2004Q1 and 2015Q4.

The parameters chosen outside the model are summarized in Table (3). Table (4) shows the parameters chosen jointly in the moment matching exercise. Details on data sources are in the appendix.

5.2 Model fit

To evaluate model performance we compare simulated model statistics to their empirical counterparts.[19] We simulate the model for 5000 periods, discarding the first 100 to eliminate the effects of initial conditions, and calculate statistics from the simulated model data.[20] Table (5) compares model and data statistics that were not targeted in the calibration. Model and actual data are treated symmetrically, that is both are filtered using an HP-filter, and output, consumption and hours are in logs. The data sources are mostly the OECD or Banco de Mexico, and are discussed in detail in the appendix.

The Table shows that the model is successful at matching aspects of public finance in

[18] Chetty et al. (2011) for example report a value of 2.84 from a meta-analysis.

[19] In the appendix we also look in more detail at equilibrium policy functions for interest rates, tax rates, inflation and seigniorage.

[20] Longer simulation samples do not significantly change the results.

Table 5: Simulations results: Calibrated model vs. data

		Data	Model
Means	Domestic nominal interest rate	0.075	0.140
	External spread	0.019	0.036
	Tax revenue/GDP	0.120	0.113
	Seigniorage/GDP	0.004	0.003
	External public debt service/GDP (annualized)	0.024	0.052
Correlations with GDP	Domestic nominal interest rate	-0.29	-0.25
	External spread	-0.65	-0.57
	Tax rate	-0.17	-0.67
	Seigniorage/GDP	-0.73	0.68
	External debt service/GDP (annualized)	0.25	0.47
	Inflation	-0.40	-0.04
Volatilities	Domestic nominal interest rate	0.0057	0.0443
	External spread	0.0066	0.0245
	Tax revenue/GDP	0.0118	0.0315
	Seigniorage/GDP	0.0002	0.0048
	External public debt service/GDP (annualized)	0.0056	0.0100
	Inflation	0.0067	0.1318

Mexico and its macroeconomy more generally. Overall, it does better at capturing first moments of the data, as well as its cyclical properties, than at second moments. It predicts that the average yield on nominal, domestically held bonds 14.4%, compared to 7.5% in the data, and that the spread on external bonds is 2.8% compared to 1.9%. The model implied rate is higher than the data counterpart due to our calibration of the risk free rate to the return to capital, which is higher than the average rate on nominal bonds for the time period considered.

In terms of public finance, the model correctly predicts that the majority of government revenue comes from labor taxation at 11% of GDP compared to 12% in the data. Very little comes from seigniorage at less than half a percent of GDP in both the data and the model.[21] The model overpredicts public debt service compared to the data, at just over 5% of GDP compared to 2.4%.[22]

The model does well at capturing the cyclical aspects of the data, as shown in the second

[21]Even in highly indebted states of the world with below average productivity realizations this does not rise above 20%. See the appendix for plots of seigniorage as a function of the state.

[22]Borrowing flows and debt stocks are too closely related in any one period debt model compared to the data where borrowers do not roll over their debt stocks every period. There is thus a tension whether to evaluate model predictions of stocks or flows. We choose to compare flows (debt service) rather than stocks given that flow revenues and their relative contributions to the government budget are important. The reason we focus on external public debt service is that total public debt service, or more generally net bond revenues, are less reliably measured in the data.

section of Table (5). It successfully predicts countercyclical domestic interest rates, external spreads, tax rates and inflation, although it underpredicts the degree to which inflation moves against the cycle. Interest, inflation and tax rates are higher in bad times as the government struggles to raise revenue from sources other than bonds which become too expensive. The model predicts procyclical rather than countercyclical seigniorage. The reason for this, as well as the weak countercyclicality of inflation in the model relative to the data, is that when default risk is sufficiently high the government cannot generate ever more seigniorage revenue because household marginal utility and thus money demand drop in default.[23]

The final section of Table (5) compares volatilities of the same variables in the model and the data. For the calibrated output volatility, the model variables are too volatile relative to their data counterparts. The period that the model is calibrated to is one of unprecedented stability in the Mexican economy. We chose to calibrate to post-2004 for reasons of availability of portfolio shares, but it means that both the averages - see for example the interest rates in the first section of Table (5) - and volatilities of most macro variables are historically low. The model cannot capture this when calibrated to match the low inflation rate and output volatility that was observed over this time period.

5.3 The role of debt denomination and ownership

In this section we use the model to study the role that debt denomination and ownership play in determining macroeconomic outcomes. We first show how the government's ability to raise revenue and simulated equilibrium outcomes vary with the structure of its portfolio, then compare equilibrium outcomes for 4 common types of portfolios, test the predictions of the model, and finally analyze the welfare implications.

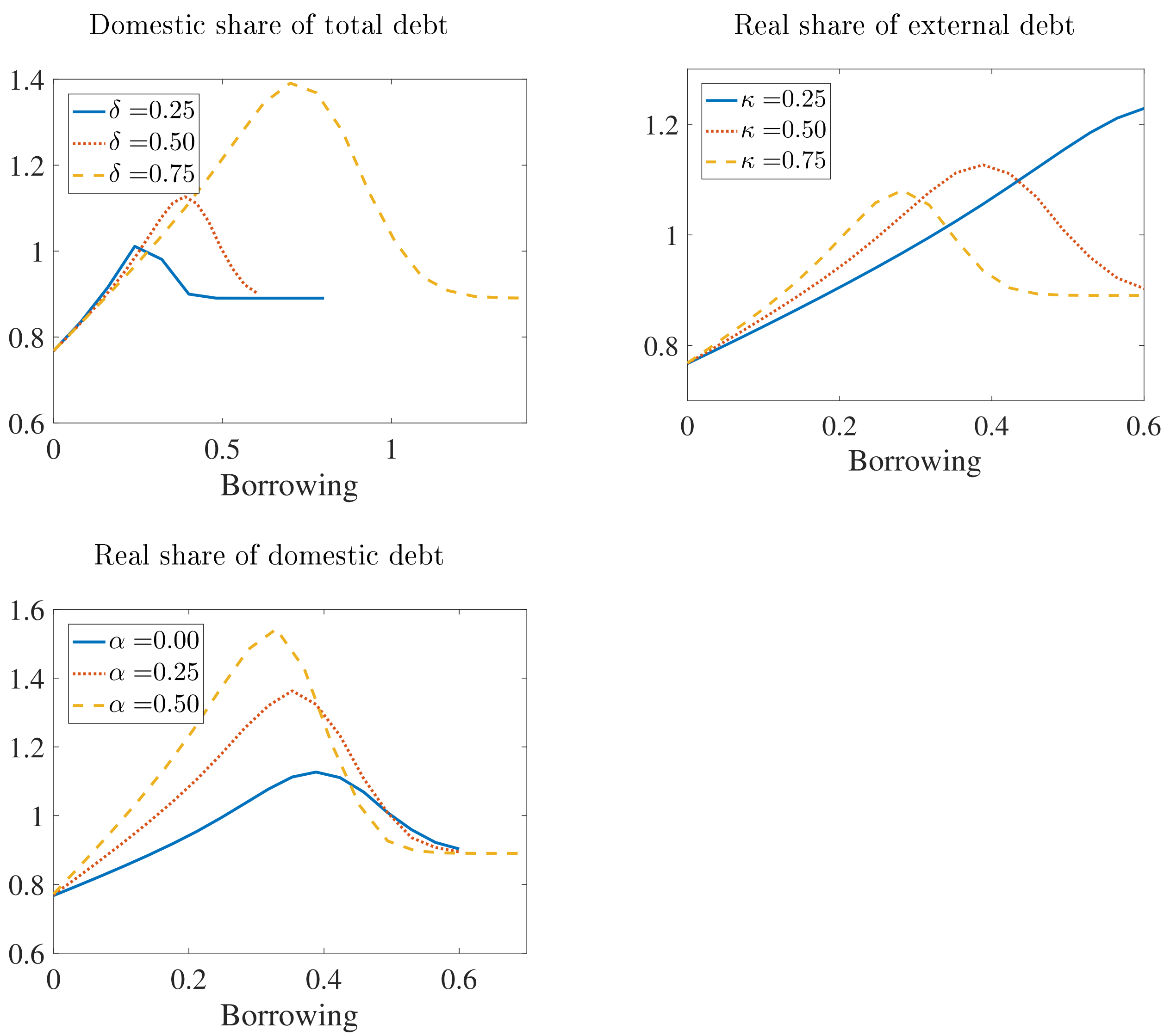
5.3.1 Laffer curves across portfolios

We can compare government revenues as a function of borrowing for a range of different portfolios to illustrate how the debt composition affects incentives. This is shown in Figure (3). Each curve is the sum of money and both types of bond revenues, that is $\zeta_1^1(z,b') + [\delta\zeta_2^1(z,b') + (1-\delta)\zeta_3^1(z,b')]b'$, and all are plotted for mean productivity realizations.

The comparison shows that domestic debt tends to increase debt accumulation while real debt tends to shift incentives towards outright default away from inflation. Moreover, the two forces interact: When the real share of domestic debt rises, the domestic ownership mutes much of the effect of the denomination change.

[23] See the policy functions in the appendix for a graphical illustration of this.

Figure 3: Laffer curves across portfolios



Panel a: $\alpha = 0, \kappa = 0.5$. Panel b: $\alpha = 0, \delta = 0.5$. Panel c: $\delta = \kappa = 0.5$

The first result can be seen in the top left panel of Figure (3) which plots the Laffer curve for economies with different shares of domestic debt δ. It shows that as δ is increased, Laffer curves remain upward sloping for wider ranges of borrowing. This reflects default risk kicking in at higher borrowing levels. The increasing share of debt that is held at home reduces default incentives and in equilibrium enables the government to extend the range of borrowing that carries no default risk. The slope of the Laffer curve is slightly convex prior to the peak. This reflects the positive contribution of revenues from money printing which increases towards the peak of the Laffer curve (recall panel (c) in Figure (2)).

The top right panel of Figure (3) shows that increasingly real debt tends to shift incentives towards outright default. It plots the Laffer curve for varying real external shares κ. The peak of the curve is at lower levels of borrowing for higher κ: When a larger share of debt is real, the ability of the government to adjust its debt burden by means of inflation is lower, so it shifts towards outright default. The Laffer curve also becomes steeper when the real share of external debt κ is higher: For the same face value of borrowing the government generates higher revenue because marginal bond revenues are less affected by inflation risk.

The bottom panel of Figure (3) finally shows the effects of varying the real share of domestic debt α. Unlike in the case of higher κ, the Laffer curve barely shifts inwards when default becomes the more effective policy tool: More real debt, if it is held domestically, does not tilt incentives towards default as much since domestic default is costly. At the same time, the curve becomes steeper as α rises: As in the case of higher external real shares, the government generates more revenue as inflation premia depress bond prices less. This factor is compounded here by the fact that when the debt is held domestically, marginal utilities also contribute to higher revenues (recall Figure (2)).

5.3.2 Equilibrium outcomes across portfolios

To look at the equilibrium differences across economies with different portfolio structures, we compare model data from the same set of economies that we considered in section (5.3.1). Table (6) presents the summary statistics, with three main results:

First, a higher share of domestic debt δ leads, as suggested by the Laffer curve discussion, to higher debt to GDP ratios in equilibrium. The differences are not dramatic with a change from 5.7% to 8.2% when raising δ from one to three quarters. Nonetheless, default rates rise in equilibrium. The effect on default rates is ambiguous in principle: Everything else equal, default is less appealing when the government borrows to a larger extent at home, which makes it cheaper and thus encourages debt accumulation. But if this latter force is too strong, it may in equilibrium more than offset the weaker default incentives, as it does here. Consistent with this, inflation increases from -4% to 17%.

Table 6: Simulation results: The effect of denomination and ownership

		Debt/GDP	Default rate	Inflation	Tax rate	$\rho(\pi, \mathrm{GDP})$	$\rho(\tau, \mathrm{GDP})$
(a)	$\delta = 0.25$	0.057	0.016	-0.044	0.120	0.08	-0.55
	$\delta = 0.50$	0.065	0.017	0.012	0.114	-0.01	-0.64
	$\delta = 0.75$	0.082	0.019	0.170	0.102	-0.23	-0.78
(b)	$\kappa = 0.25$	0.062	0.015	0.041	0.111	-0.08	-0.68
	$\kappa = 0.50$	0.065	0.017	0.012	0.114	-0.01	-0.64
	$\kappa = 0.75$	0.064	0.019	-0.008	0.116	0.04	-0.60
(c)	$\alpha = 0.00$	0.065	0.017	0.012	0.114	-0.01	-0.64
	$\alpha = 0.25$	0.079	0.021	0.071	0.110	-0.23	-0.77
	$\alpha = 0.50$	0.090	0.023	0.132	0.106	-0.37	-0.83

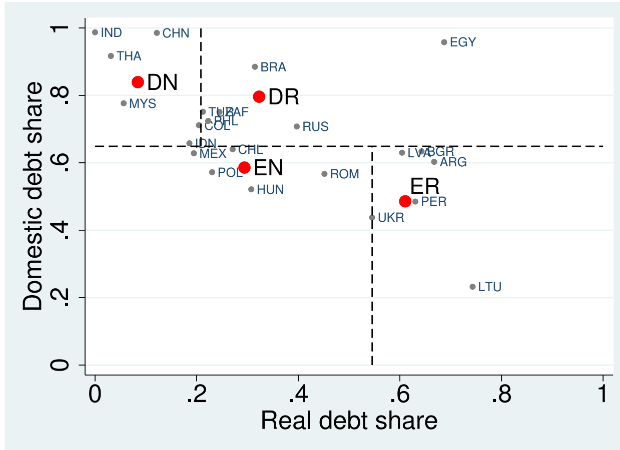
a: $\alpha = 0, \kappa = 0.5$. b: $\alpha = 0, \delta = 0.5$. c: $\delta = \kappa = 0.5$. Default and inflation rates are annual. π: inflation, τ: tax rate

Second, increasing the real share of external debt leads as expected to a switch out of inflation and into default. When κ rises from one to three quarters, inflation drops from 4% to virtually zero, while default rates increase from 1.5 to 1.9%. At the same time, inflation becomes less countercyclical: The government does not find it optimal to use the printing press as a revenue generator in bad times or just prior to default since it would require much more of it to create substantial revenues when the real debt burden is less affected by inflation.

Third, and perhaps most interestingly, increasing the real share of domestic debt leads to higher, not lower, inflation. Inflation increases from around 1% when none of the domestic debt is real to 13% when half of it is. At the same time, default rates rise from 1.7% to 2.3% and debt to GDP ratios from 6.5% to 9%. It is the combination of costly default on domestic households together with a low inflation tax base that leads to this results. Incentives to default do not rise as much as they would if the denomination of external debt was shifted, thus encouraging more debt accumulation. And equilibrium debt is sufficiently high for both default and inflation rates to be higher as well. The government is forced to use inflation to finance its relatively high debt burden, especially in bad times as shown by the stronger countercyclicality of inflation.

These results highlight that it is debt denomination and ownership jointly that matter for equilibrium crisis risk and inflation outcomes. The model cautions against looking at denomination in isolation when considering portfolio management policies. Instead, the implications of borrowing in nominal terms depend on who holds the debt.

Figure 4: Four portfolio types - Categorization



5.3.3 Four typical portfolios

To investigate how much macroeconomic outcomes are likely to vary across debt structures that we see in the data, we identify four "typical" portfolios and compare statistics from model economies with these compositions. To identify the portfolios, we start from cross-section debt characteristics, that is their average domestic and real debt shares for each country. We then assign each country to one of four types: Domestic Real (DR), Domestic Nominal (DN), External Real (ER) or External Nominal (EN), according to the chief characteristic of their debt portfolio: Countries above the median domestic debt share across countries are categorized as Domestic, and as External otherwise. Within Domestic and External, respectively, countries above the median share of real debt across countries are then categorized as Real, and Nominal otherwise.[24] Figure (4) illustrates the way the portfolios are constructed and what countries are included in each category.

The resulting model statistics and portfolio compositions are reported in Table (7). The key result shown in the Table is that ownership characteristics result in larger macroeconomic differences than denomination characteristics, especially for inflation and also debt: The economies grouped by ownership are more similar than grouped by denomination. For example, a typical Domestic debt economy has more debt and higher inflation than one with largely External debt. Inflation rates in columns one and two are 35 and 43% respectively, whereas they are only around 4% in both columns three and four. Moreover, more extensive use of inflation finance in Domestic debt economies goes hand in hand with more counter-

[24] The implied portfolio characteristics of each group change either not at all or only slightly if we (i) first group by denomination and then by ownership, (ii) define the real debt share based on the direct measure x (see the appendix) rather than the weighted average, (iii) do any of this in the pooled data set rather than the cross-section.

Table 7: Four portfolio types - Simulation results

	Domestic		External	
	Real	Nominal	Real	Nominal
Debt/GDP	0.105	0.102	0.075	0.071
Default rate	0.029	0.023	0.024	0.020
Inflation	0.353	0.439	0.036	0.044
Tax rate	0.091	0.084	0.114	0.112
ρ(Inflation,GDP)	-0.46	-0.44	-0.16	-0.09
ρ(Tax rate,GDP)	-0.85	-0.83	-0.74	-0.71
δ	0.79	0.85	0.49	0.59
κ	0.63	0.36	0.85	0.64
α	0.24	0.04	0.36	0.06

cyclical tax and inflation rates. The differences for debt are not as large but noticeable, with the Domestic economies' debt to GDP ratio around 10% compared with approximately 7% in the External cases.

The picture is less clear cut for default rates. Conditional on denomination, default rates are higher in the Domestic economies where debt ratios are higher. Conditional on ownership, they are higher in the Real economies where inflation is relatively less effective. But the differences are not as large as in the case of inflation, with the lowest default rate 2% and the highest 2.9% annually.

5.3.4 Testing the model predictions

The previous sections identified two testable predictions of the theory. First, the effect of debt denomination on inflation depends on ownership: Real domestic debt raises inflation while real external debt lowers it. And second, the effects of ownership on macroeconomic outcomes, including inflation and debt, are larger than those of denomination.

To test these, we run regressions of the form

$$y_{it} = \boldsymbol{\beta x_{it}} + \boldsymbol{\gamma z_{it}} + \epsilon_{it} \tag{28}$$

where explanatory variables are either inflation π_{it} or the debt to GDP ratio b_{it}/y_{it}, and regressors are either the real shares of external and domestic debt, $\boldsymbol{x_{it}} = (\alpha_{it}, \kappa_{it})$, or the domestic and real shares of debt, $\boldsymbol{x_{it}} = (\delta_{it}, \omega_{it})$. We use as the measure of the real debt share not the weighted average which would involve the domestic share itself, but instead the direct measure from the data.

Our observations are the panel of 24 countries from 2004Q1 through 2015Q4 for which we have portfolio data. We estimate equation (28) using panel-feasible generalized least squares

Table 8: Regression results

	π	π	$\frac{b}{y}$	$\frac{b}{y}$	π	π	$\frac{b}{y}$	$\frac{b}{y}$
α	0.027***	1.157***	0.080***	0.046				
	(0.007)	(0.373)	(0.023)	(0.038)				
κ	-0.004	0.090	-0.177***	-0.131***				
	(0.005)	(0.249)	(0.011)	(0.023)				
δ					0.057***	1.651***	0.306***	0.099*
					(0.008)	(0.382)	(0.026)	(0.051)
ω					0.033***	0.975***	0.062***	0.081
					(0.007)	(0.357)	(0.021)	(0.050)
Controls	No	Yes	No	Yes	No	Yes	No	Yes
N	886	118	970	135	930	121	1037	138

Standard errors in parentheses. $*p < 0.10, **p < 0.05, ***p < 0.01$.

(FGLS) with heteroskedasticity across panels.[25] As controls we include real PPP-adjusted per capita GDP, a measure of exchange rate regime and a measure of monetary policy regime, including whether the country is an inflation targeter. We report two sets of results, one at quarterly frequency without controls since these are not available at high frequency, and one at annual frequency with controls.[26]

Real domestic debt raises inflation We first investigate the relationship between the real shares of external and domestic debt, α and κ, and inflation. The model predicts that we should find that real domestic debt is positively related to inflation, while real external debt is negatively related to it: $\hat{\beta}_1 > 0$ and $\hat{\beta}_2 < 0$. The results are reported in the first two columns of Table (8). They show that the model prediction is broadly consistent with the data. The coefficient on real domestic debt shares is positive and significant at the 1% level. The effect of real external debt shares is imprecisely estimated.[27]

To check whether there is support for the mechanism that the model suggests, we estimate the relationship between the real shares of external and domestic debt, α and κ, and debt to GDP ratios. The model predicts that inflation is high whenever the domestic real debt share

[25] We do not use fixed-effects regressions since the variation in portfolios within countries is not large enough, and instead control for other variables that may affect the estimated relationship. See the appendix for results based instead on pooled OLS with clustered standard errors that are qualitatively similar but less precisely estimated.

[26] See the appendix for details of data sources.

[27] Looking at the underlying data, this can in part be attributed to Argentina and the Ukraine which in our sample have both high inflation and high external debt shares. The same regressions with a dummy variable for observations from those two countries yields more precisely estimated and negative point estimates for the effect of real external debt shares on inflation.

is high precisely because it allows the government to accumulate relatively more debt, and vice versa for the real external debt. We therefore expect $\hat{\beta}_1 > 0$ and $\hat{\beta}_2 < 0$. The results from this specification are in columns three and four of Table (8). They corroborate the theory: Real external debt shares are significantly negatively correlated with debt to GDP ratios, while the effect of real domestic debt shares is positive, although not significantly in the annual regressions.

Ownership is more important than denomination A second testable prediction of the model is the relative importance of ownership over denomination in determining inflation rates and debt levels. To investigate this empirically, we regress inflation and debt to GDP ratios on the domestic debt share and the real debt shares, δ and ω. Results are reported in columns 5 through 8 of Table (8).

They provide some evidence that domestic debt shares are more strongly linked to inflation and debt ratios than real debt shares, as in the theory. The difference in the coefficients is significant at the 1% level for the quarterly regressions, but not in the annual regressions even though the point estimates are consistent with the theory qualitatively.

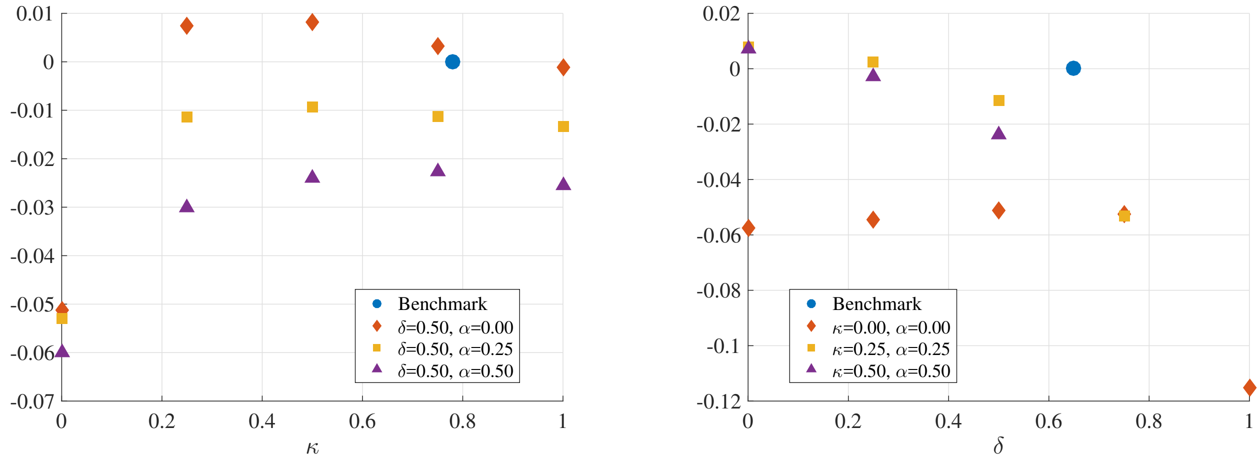
5.3.5 Welfare

What portfolio structure is desirable from a normative standpoint? We compute the consumption equivalent ex-ante welfare gains of a range of alternative economies relative to the benchmark, and find that nominal and external debt tends to improve welfare. [28]

Figure (5) plots welfare gains for a range of economies with different portfolio structure relative to the benchmark to illustrate this result. The left panel shows the effects of varying the real shares of both domestic and external debt. Real domestic debt α unambiguously deteriorates welfare, while an interior real external share κ of around 25% is optimal. It is preferable to hold some small share of external debt in real terms because fully nominal external debt encourages inflation too much. With debt entirely nominal and external, the equilibrium outcome would be for debt levels to fall and inflation thus not to increase substantially, but lowering the nominal share of external debt a little removes incentives to inflate and allows the government to accumulate more debt at similar inflation and default rates. More generally, the welfare effects of nominal debt depend on the flexibility gains versus inflation costs. On the one hand, nominal debt provides flexibility since unexpected

[28]Gains are defined as the percentage of permanent credit good consumption that households in the benchmark economy would be willing to give up to live in an alternative economy: $E_0 \sum_t \beta^t u(c_{1A}, c_{2A}, n_A) = E_0 \sum_t \beta^t u(c_{1B}, c_{2B}(1+\Delta), n_B)$. We report the ex-ante welfare gain, that is the unconditional expectation of the gain with zero assets.

Figure 5: Welfare gains



inflation lets the government fine tune the debt burden. On the other, it introduces incentives for distortionary inflation. Here, the flexibility gains tend to dominate.

The left panel of Figure (5) plots welfare gains associated with varying the domestic share of debt, for a given range of denominations. It shows that external debt is optimal. For entirely nominal debt an interior domestic share of around 75% is optimal, but otherwise the optimal domestic share falls to zero, and the entirely nominal internal stock is associated with welfare losses relative to an all external portfolio. In terms of intuition, domestic debt provides disincentives to expropriate, since it is costlier to default on your residents, but this allows the government to borrow more, and thereby create costly inflation and default. This second effect dominates here, so that domestic debt tends to entails welfare losses. The first effect dominates for sufficiently low ranges of domestic debt shares but only if the entire debt stock is nominal. In that case the inflation that comes with higher debt levels is less detrimental as it applies to the entire stock of debt and can be used effectively to fine tune.

6 Conclusion

Whether emerging market government debt pays in nominal or real terms and who holds it matters for crisis risk. In this paper we have characterized and quantified the role that debt denomination and ownership play for default and inflation outcomes by incorporating realistic debt portfolio structures into a dynamic general equilibrium model of sovereign borrowing without commitment. We have used the calibrated model to sharpen our understanding of the effects of shifts in the portfolio structure. Our key conclusions, which find support in the data, are that domestic debt ownership, and especially real domestic debt, is an important driver of inflation; that ownership is the more important factor affecting inflation and debt levels; and that nominal and external portfolios tend to be welfare improving since they take advantage of the flexibility benefits of inflation without encouraging too much debt accumulation.

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Appendix

A Model Appendix

A.1 Deriving the implementability constraint

Write the government budget constraint (11) recursively as

$$\frac{1}{p} + (1-d)\delta\left(\frac{1-\alpha}{p}+\alpha\right)b + g - \tau wn + x = \frac{1+\mu}{p} + (1-d)\delta q\left(\frac{(1-\alpha)(1+\mu)}{p}+\alpha\right)b'$$

where we have imposed money market clearing and x is defined as in (15). Then substitute for μ and q using (5) and (6), and use the definitions of ζ_1^1 and ζ_2^1 from the main text to get

$$u_2\left[\frac{1}{p} + (1-d)\delta\left(\frac{1-\alpha}{p}+\alpha\right)b + g - \tau wn + x\right] = (1-d)\left(\zeta_1^1(z,b') + \delta\zeta_2(z,b')b'\right) + d\zeta_1^0(z)$$

Now substitute for the tax rate using (4)

$$u_2\left[\frac{1}{p} + (1-d)\delta\left(\frac{1-\alpha}{p}+\alpha\right)b + g - y + x\right] - u_3 n \quad = \quad (1-d)\left(\zeta_1^1(z,b') + \delta\zeta_2(z,b')b'\right) + d\zeta_1^0(z)$$

ans use market clearing (14) and 13 to arrive at (16). This constraint is the same as in a closed economy (except that the debt stock and borrowing are scaled by δ), external savings only enter explicitly through the resource constraint.

A.2 Deriving the government Euler condition

Recall that we denote by λ_1 and λ_2 the Lagrange multipliers on the implementability constraint (16) and the resource constraint (21), let $\mathcal{Z}(b)$ be the set of z for which the government does not default given b, and let $\hat{\zeta}_i^1(z,b)$ be defined such that $\zeta_i^1(z,b') = E_{z'|z}[\hat{\zeta}_i^1(z',b')], i = 1,2,3$.

FOC with respect to b':

$$\lambda_1\left(\frac{\partial\zeta_1^1(z,b')}{\partial b'} + \delta\zeta_2^1(z,b') + \delta b'\frac{\partial\zeta_2^1(z,b')}{\partial b'}\right) + \\ \lambda_2(1-\delta)\left(\zeta_3^1(z,b') + b'\frac{\partial\zeta_3^1(z,b')}{\partial b'}\right) + \beta E_{z'|z}\left[\frac{\partial V^1(z,b')}{\partial b'}|z' \in \mathcal{Z}(b')\right] = 0$$

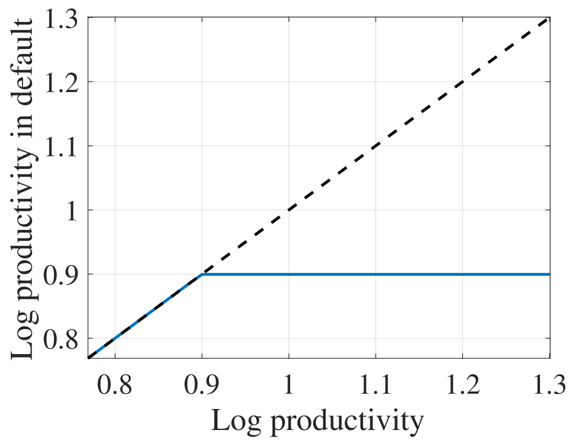
Envelope condition:

$$\frac{\partial V^1(z,b)}{\partial b} = -\lambda_1 \delta \hat{\zeta}_2^1(z,b) - \lambda_2(1-\delta)\hat{\zeta}_3^1(z,b)$$

Combining these by substituting out the value function yields expressions (26).

A.3 Default cost function

Figure 6: Default cost function

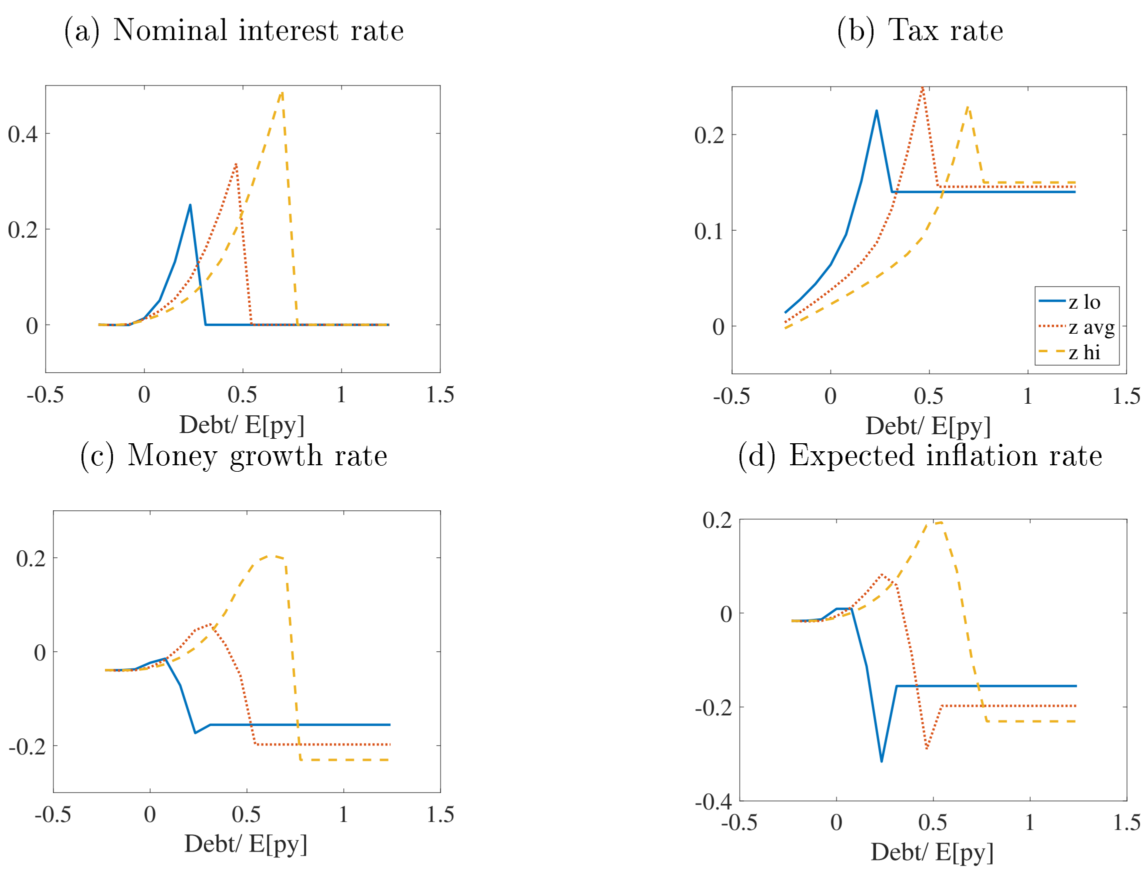


A.4 Policy functions

Equilibrium interest, tax and money growth rates We can look at some of the model implied equilibrium policies as a function of the state to build intuition for the mechanics of the model. Figure (7) plot labor income tax rates, the domestic bond yield (equivalently, the nominal interest rate since the share of real domestic debt is zero in the calibration), expected inflation and the money growth rate. The latter three are all annualized. All rates depend not just on the state but also on the borrowing decisions b', and we have imposed that the optimal borrowing policy is being followed, that is $b' = \boldsymbol{h}(z,b)$.

The Figure shows that all rates are higher the higher debt and the lower productivity is, conditional on no default. Recall that when debt is high or productivity low, default risk is high in the model. This in turn means that marginal bond revenue is low as default risk is priced in, and so servicing the debt is costly. This is reflected here in the nominal interest rate spiking in these high debt- low productivity states of the world. The government therefore optimally responds by increasing tax and money growth rates, which results in increased expected inflation rates. Both money growth and expected inflation rates, rather than increasing monotonically with debt, level off and fall at debt levels just shy of default

Figure 7: Equilibrium rates and revenues as a function of the state



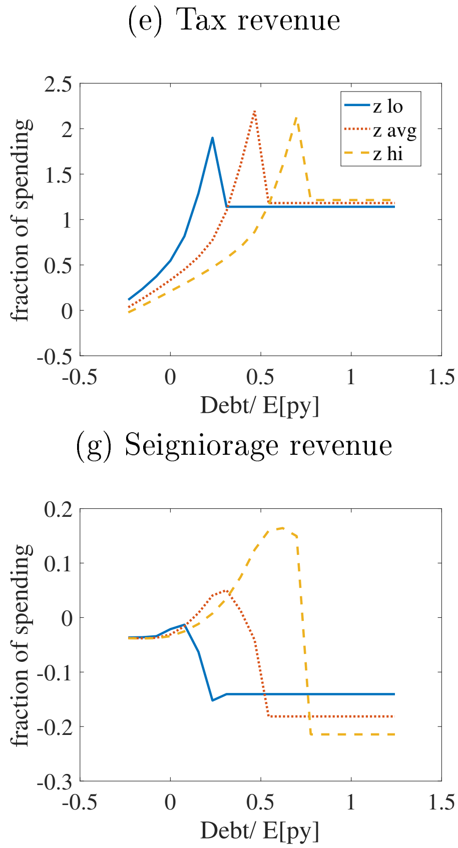
(a) Nominal interest rate

(b) Tax rate

(c) Money growth rate

(d) Expected inflation rate

Figure 8: Equilibrium sources of public finance as a function of the state



(e) Tax revenue

(f) Net bond revenue

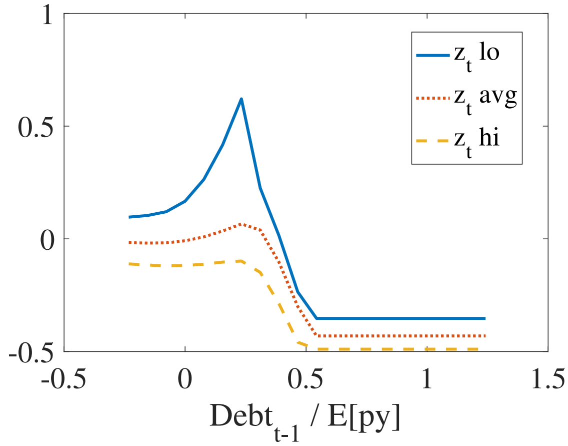
(g) Seigniorage revenue

as they incorporate the expectations of lower debt service and hence less need to use the printing press in default.

Public finance Figure (8) illustrates how the government's sources of public finance vary with the state. It plots the contributions of its three sources of revenue which together add up to the fixed level of government spending: Labor income taxes, seigniorage and net borrowing as a share of government expenditures, as a function of the state and again with borrowing following the equilibrium rule $b' = \boldsymbol{h}(z, b)$. We can see that as debt rises or productivity falls, the government finances itself increasingly with taxes and seigniorage. Bond finance becomes too expensive since interest rates rise with inflation and default risk. Quantitatively, seigniorage makes a relatively minor contribution to public finance of at most 10% of expenditures at average productivity levels. Labor income taxes make up the bulk of the revenue throughout. At low levels of debt, bond revenue contributes more than 50%, but this quickly falls as default and inflation risks rise.[29]

[29]Note that this is not a statement about the contributions observed in simulations, but as a function of the state. The simulation results show that equilibrium financing shares are empirically plausible.

Figure 9: Realized inflation rate $\pi_t = \frac{\bar{P}_t}{\bar{P}_{t-1}} - 1$, with $z_{t-1} = E[z]$



Inflation We can finally also look at realized inflation as a function of current and past fundamentals. Recall from equation (27) that realized inflation between t and $t-1$ depends not just on the state in $t-1$, but also the productivity realization in t. Figure (9) therefore plots equilibrium inflation $\pi_t \equiv \frac{\bar{P}_t}{\bar{P}_{t-1}} - 1$ as a function of debt outstanding b_{t-1} for a range of productivity realizations z_t, conditional on productivity being at its long run mean the previous period, $z_{t-1} = E[z]$, and with borrowing following the equilibrium rule $b_t = \boldsymbol{h}(z_{t-1}, b_{t-1})$. The rate is expressed in annualized terms.

The Figure shows that the government optimally inflates when productivity turns out to be worse than in the previous period (in this case, below average) and debt is sufficiently high but not high enough for outright default. If debt is high but productivity improved relative to yesterday and is now above trend, the government avoids very high inflation rates. The reason for this is again that its ability to raise revenue from bonds varies with productivity. In good times, bond finance is relatively cheap as well as non-distortionary, so the government uses that instead of inflation to balance its budget. In bad times it cannot raise as much revenue from borrowing due to inflation and default risk premia. As a result, it shifts its financing to inflation, as long as debt is not so high that default is imminent. The Figure suggests that the model may be able to account for countercyclical inflation finance in simulations, with inflation spiking when productivity is below average, for a given level of debt.

B Data Appendix

B.1 Portfolio data

The data set is available at http://www.imf.org/external/pubs/ft/wp/2014/Data/wp1439.zip. We use the following variables with corresponding sheet and row in the data set spreadsheet:

- (GGall) General government debt: Table 1, 1:25
- (ExtGGall) External general government debt: Table 2, 1:25
- (GG) General government debt securities: Table 1, 27:52
- (ExtGG) External general government debt securities: Table 2, 53:78
- (LCCG) Local currency central government debt securities: Table 1, 54:79
- (ExtLCCG) External local currency central government debt securities: Table 2, 107:132
- (GGy) General government debt to GDP: FX, 27:51

Foreign holdings of general government debt exclude foreign official loans. Data on foreign ownership of local-currency central government debt securities are available for most countries in the sample from national data sources. Like Arslanalp and Tsuda (2014), we use this as a proxy for foreign ownership of local-currency general government debt securities, assuming that foreign holdings of local government debt securities are small.

The authors do not discuss how local currency indexed debt is classified. Assuming that it is treated as local currency debt, our estimates for α and κ are lower bounds. Du and Schreger (2015) who construct similar measures of government debt portfolio shares note that, where available, nonresident holdings holdings of indexed debt are very small relative to nonresident holdings of local currency debt, suggesting that our estimate of κ is unlikely to be a substantial underestimate. Similarly for domestic real debt shares, indexed bond markets in general tend to be small relative to non-indexed so α is unlikely to be large and certainly smaller than κ.

B.2 Inferring portfolio shares

Consider the following stylized representation of the sovereign's debt portfolio:

	Nominal	Real	Nom+Real
Domestic	$\delta(1-\alpha)$	$\delta\alpha$	δ
External	$(1-\delta)(1-\kappa)$	$(1-\delta)\kappa$	$1-\delta$
Int+Ext	x	$1-x$	1

We can obtain estimates of δ, x and the external share of nominal debt $y \equiv \frac{(1-\delta)(1-\kappa)}{x}$ from the database as follows. The domestic debt share is computed as $\delta = 1 - \frac{ExtGGall}{GGall}$. The nominal share of debt is approximated as $x = \frac{LCCG}{GG}$. Ideally we would like to know total central government debt securities for the denominator, but these are not available in the database. This is a good approximation as long as local government debt is small relative to general government debt which tends to be true in the data in most countries. The external share of nominal debt is calculated as $y = \frac{ExtLCCG}{LCCG}$.

We then use these estimates to infer α and κ: Using the definition of y,

$$\kappa = 1 - \frac{xy}{1-\delta}$$

and since from the table, $1 - \alpha = \frac{x-(1-\delta)(1-\kappa)}{\delta}$,

$$\alpha = 1 - \frac{x(1-y)}{\delta}$$

It is possible that α (and in theory also κ although this never happens in the sample) are negative when x is too large relative to y, in which case we impose the lower bound of zero.

B.3 Portfolio share time series

Figure 10: Debt portfolio structure: Domestic share of total debt δ

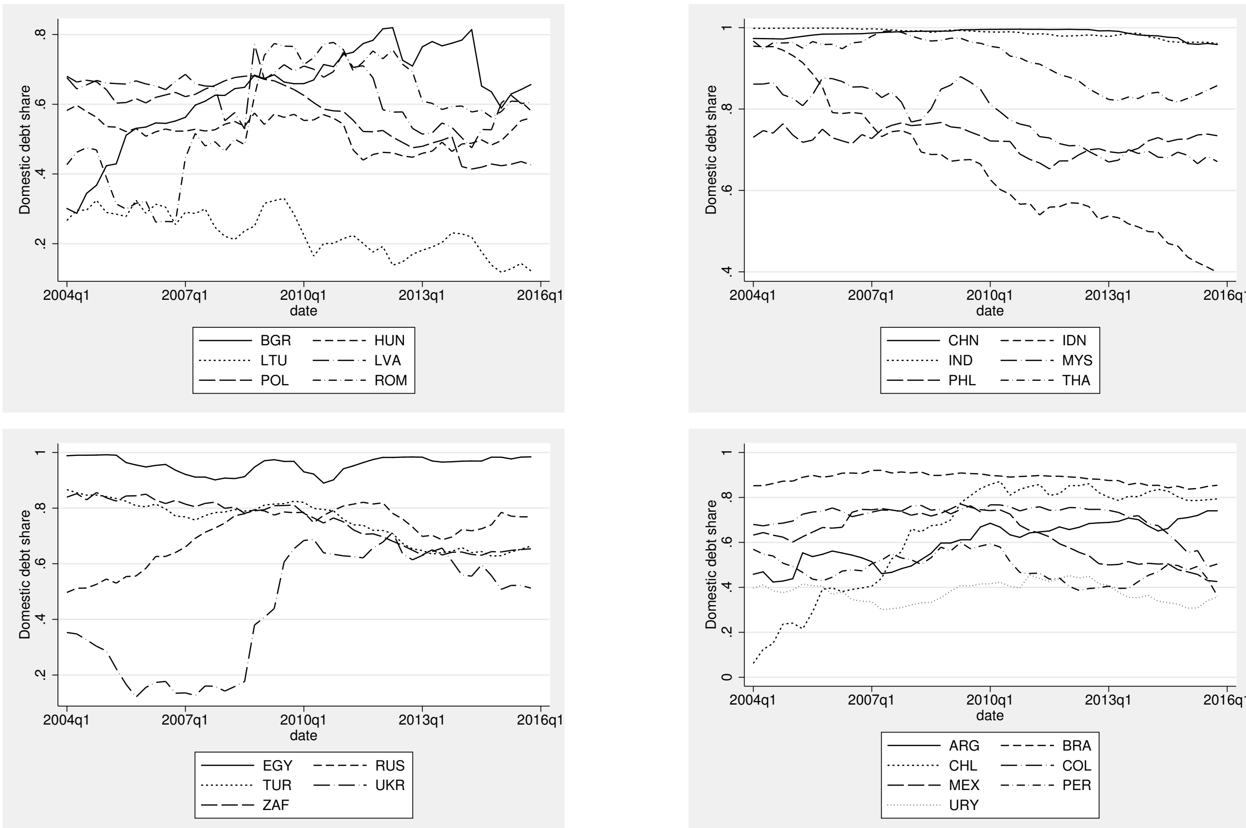


Figure 11: Debt portfolio structure: Real share of external debt κ

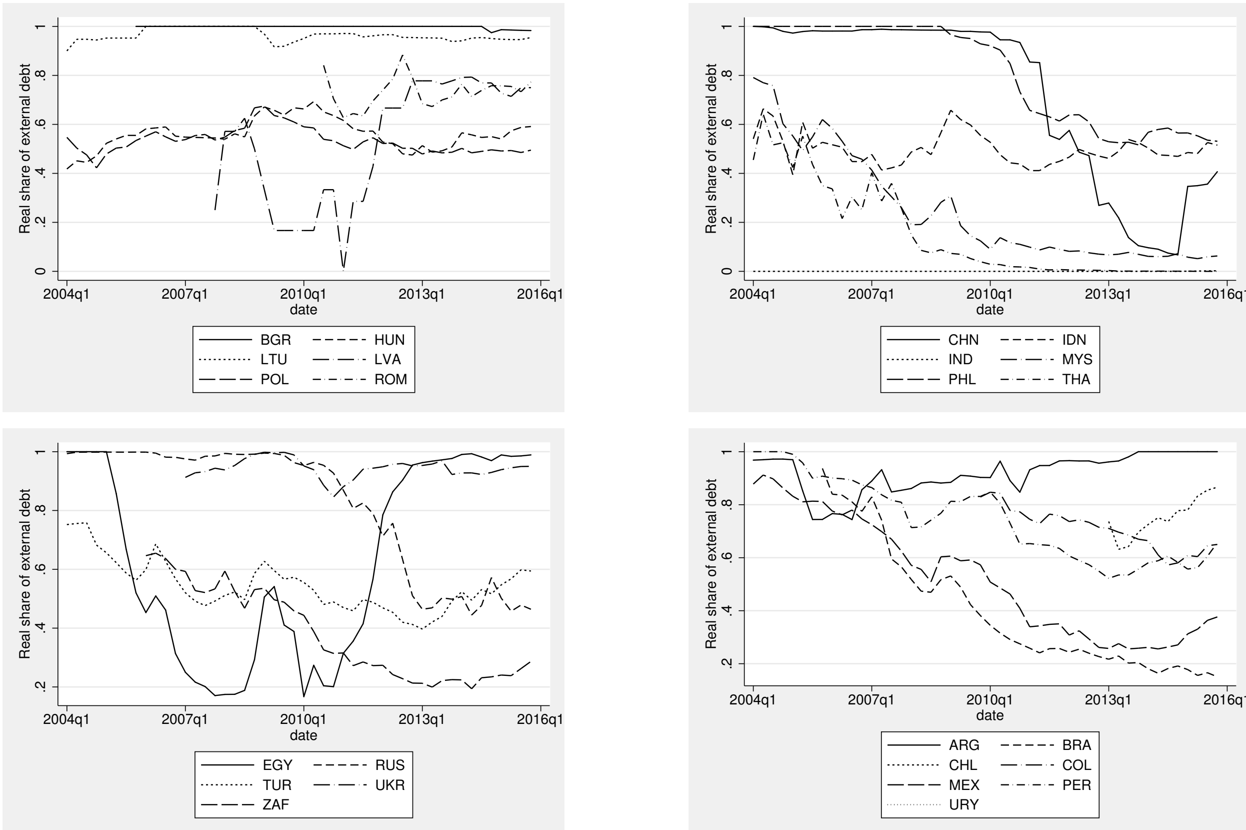
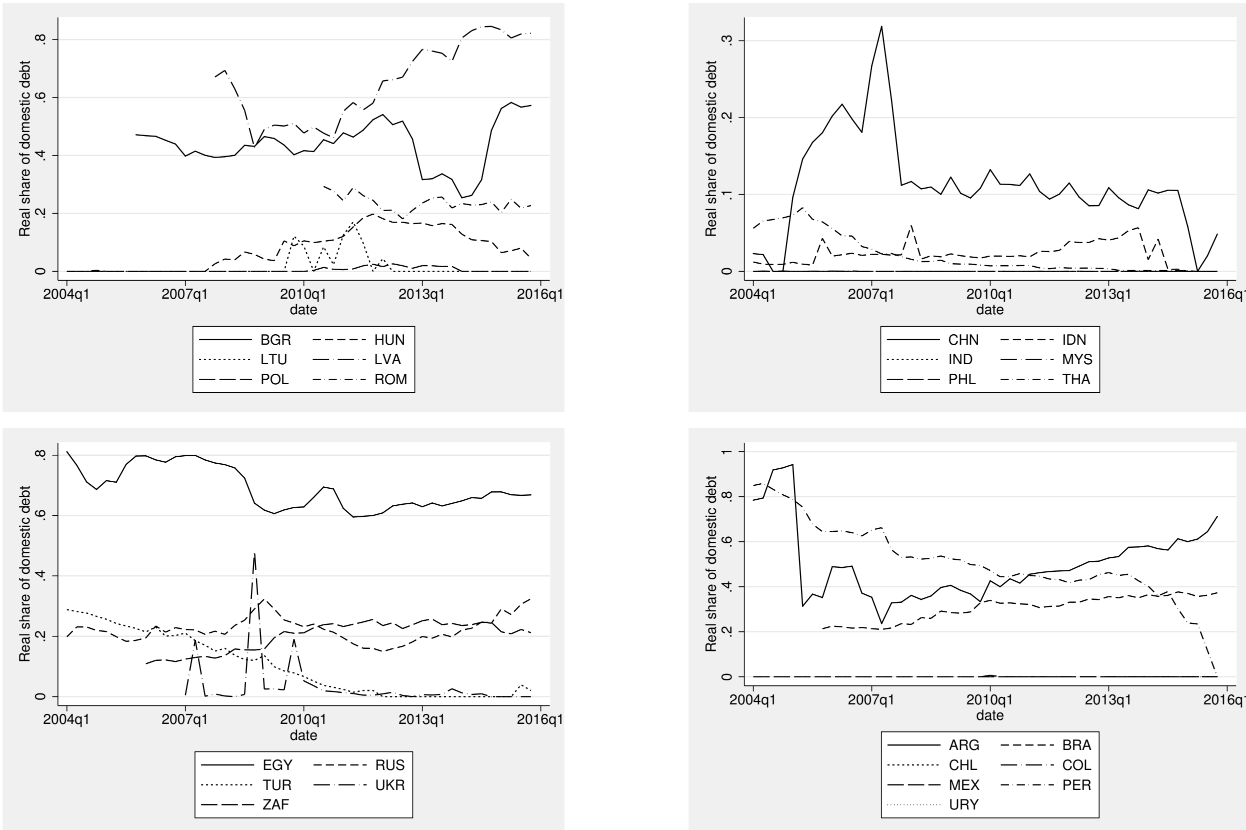


Figure 12: Debt portfolio structure: Real share of domestic debt α



B.4 Country portfolio classification

Countries are assigned to the portfolio quadrants by first splitting along the median domestic debt share, and then within that splitting by the median real debt share. The resulting classification is shown below:

	Nominal	Real
Domestic	China	Brazil
	Indonesia	Colombia
	India	Egypt
	Malaysia	Philippines
	Thailand	Russia
	Turkey	South Africa
External		Argentina
	Chile	Bulgaria
	Hungary	Lithuania
	Mexico	Latvia
	Poland	Peru
	Romania	Ukraine
		Uruguay

B.5 Data sources for calibration

In the calibration section and to evaluate the performance of the model by comparing it to the data we use the following sources.

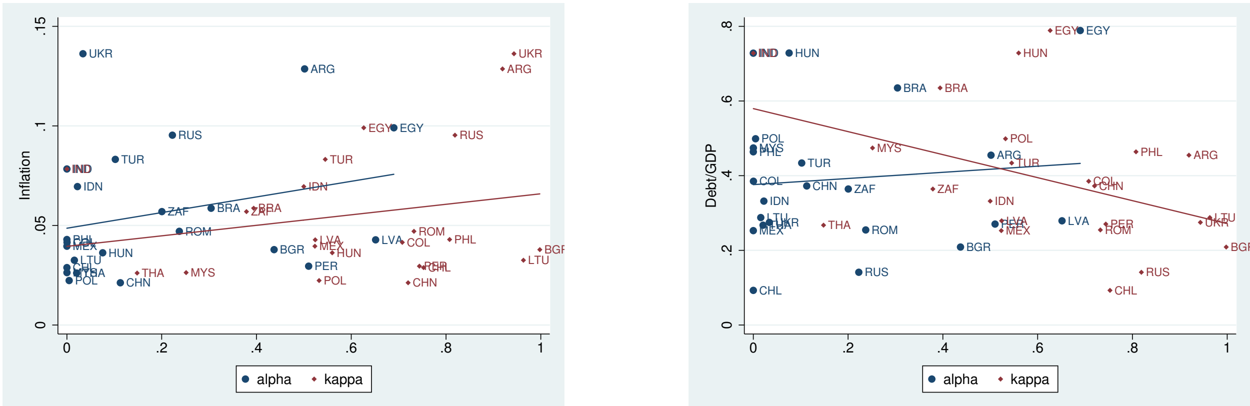
Output, consumption, government spending, net exports, CPI, the GDP deflator and M1 money stock are from the OECD, quarterly and seasonally adjusted. 1997Q1 - 2015Q2. The domestic nominal interest rate is the 1 year government bond yield on CETES bonds from the OECD MEI, available from 2001Q3 through 2015Q2. The external spread is the JP Morgan EMBI spread from 1997Q1 through 2012Q3. Tax revenue and the monetary base are from Banco de Mexico, quarterly seasonally adjusted 1997Q1 through 2015Q2, and tax revenue includes personal income taxes less consumption taxes. We do not include oil related tax revenues or corporate revenues, in line with the model abstracting from oil sectors and a fully-fledged corporate sector with capital accumulation. We calibrate to the overall government consumption to GDP ratio of the economy since extractive industries and capital increase both public revenues and investment expenditures. We effectively assume that they are revenue neutral.

B.6 Regressions: Data sources and robustness

As controls in the annual panel regressions we use:

- Log of per capita GDP at PPP-adjusted constant prices. Source: WDI. Annual 2004-2015

Figure 13: Real debt shares in the cross section



- De-facto exchange rate regimes. Source: IMF AREAER. Annual 2004-2015. Classification: 1: No separate legal tender. 2: Currency board. 3: Conventional peg. 4: Stabilized arrangement. 5: Crawling peg. 6: Crawl-like arrangement. 7: Pegged exchange rate within horizontal bands. 8: Other managed arrangement. 9: Floating. 10: Free floating.

- Monetary policy regimes. Source: IMF AREAER. Annual 2004-2015. Classification: 1: Exchange rate anchor. 2: Monetary aggregate target. 3: Inflation-targeting framework. 4: Other monetary framework.

Tables (9) and (10) show additional regression results for robustness. Table (9) shows results for the effects of real domestic (α) and external (κ) debt shares on inflation and debt ratios, respectively. Columns 1 and 3 are cross-sectional OLS regression results, columns 2 and 4 show pooled OLS results in the quarterly panel, both with clustered standard errors. The last two columns use an alternative, broader measure of debt to GDP: Instead of general government debt securities ("GG", see Appendix B.1), it uses all general government debt ("GGall"). Table (10) shows results for the effects of domestic (δ) and real (ω) debt shares on inflation and debt ratios, respectively. As in Table (9), columns 1 and 3 are cross-sectional OLS regression results, columns 2 and 4 show pooled OLS results in the quarterly panel, both with clustered standard errors, and the last two columns use the broader measure of debt to GDP. The Tables show that qualitatively the results are similar to the ones presented in the main section, but less precisely estimated.

Figure (13) plots simple scatters of the data underlying the cross-section OLS regressions, along with lines of best fit. We can see that unconditionally, real domestic debt shares are positively correlated with average inflation rates in the cross section, while real external shares are more weakly positively linked. The positive slope is driven exclusively by Argentina and Ukraine, both of which have exceptionally high external real shares and inflation rates in the sample. The plot for the relationship between debt ratios and real debt shares shows already in these unconditional scatters the link that the regressions confirm: High real domestic debt shares are associated with high overall debt levels, real external debt shares the opposite.

Table 9: Additional regression results I

	(1) Inflation	(2) Inflation	(3) Debt/GDP	(4) Debt/GDP	(5) Debt/GDP (alt)	(6) Debt/GDP (alt)
α	0.034	0.017*	0.176	0.198***	0.367*	0.432***
	(0.034)	(0.010)	(0.218)	(0.032)	(0.203)	(0.065)
κ	0.019	0.009	-0.344**	-0.220***	-0.638	-0.473***
	(0.034)	(0.008)	(0.137)	(0.019)	(0.414)	(0.053)
Observations	23	886	23	970	23	970
Adjusted R^2	0.003	0.006	0.150	0.131	0.169	0.135

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 10: Additional regression results II

	(1) Inflation	(2) Inflation	(3) Debt/GDP	(4) Debt/GDP	(5) Debt/GDP (alt)	(6) Debt/GDP (alt)
δ	0.045	0.048***	0.465	0.350***	0.909**	0.775***
	(0.041)	(0.011)	(0.290)	(0.039)	(0.391)	(0.058)
ω	0.064**	0.045***	0.073	0.098***	0.164	0.218***
	(0.029)	(0.008)	(0.211)	(0.032)	(0.264)	(0.057)
Observations	23	930	23	1037	23	1037
Adjusted R^2	0.059	0.031	0.093	0.092	0.120	0.104

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$