Inflation, default and sovereign debt: The role of denomination and ownership

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Abstract

Emerging market governments hold mixed debt portfolios: They borrow both at home and abroad, and in both nominal and real terms. This paper proposes a theory of sovereign borrowing, default and inflation that incorporates such a mixed debt structure. The government optimally uses both default and inflation to balance its budget, with the portfolio structure affecting their relative benefits, as well as incentives to accumulate debt. A calibrated version of the model can account for salient features of the Mexican economy. We use the model to characterize and quantify the effects of shifts in the portfolio composition. We find that more nominal debt does not necessarily raise inflation, and that ownership is a key driver of debt accumulation because of its effects on default incentives.

Keywords: Sovereign debt crises, government debt composition, inflation
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1 Introduction

Governments in emerging market economies hold mixed debt portfolios in terms of denomination and ownership: They borrow partially from domestic investors, partially from abroad; some bonds are nominal and thus subject to inflation risk, while others, including inflation-indexed and foreign-currency bonds, are not. At the same time their debt carries non-negligible default risk, and the question of what drives costly debt crises in these countries has been and remains an important one for international and local policymakers alike.

Who holds government debt and whether its value can be inflated away are potentially important factors driving debt crisis risk. Defaults might be less likely when the government has inflation available as a tool to reduce the real debt burden, for example, whereas it might be more likely when creditors are foreign investors rather than domestic residents. This paper therefore studies the role that the composition of the debt portfolio in terms of denomination and ownership plays in shaping macroeconomic outcomes like debt levels, default and inflation rates.

We propose a simple model of sovereign debt and default that incorporates such mixed government debt portfolios. In the model, a benevolent government issues bonds, prints money, and raises taxes in order to smooth consumption of its residents. It cannot commit to any of its policies. This lack of commitment generates positive inflation and occasional default in equilibrium. When issuing bonds and money the government internalizes how their demand depends on the level of issuances: High future levels of debt or money drive up current default and inflation risk and hence reduce demand.

The bonds that the government has access to are non-contingent and replicate the empirically observed mixed denomination and ownership structure of sovereign debt portfolios: They are partially indexed instruments, a fraction of which is held and priced abroad, while the remainder is held and priced at home. The government can default on them outright at any time, and inflation erodes the non-indexed part of their value. We treat this debt structure as fixed and exogenous and focus on studying the implications of the portfolio structure on the optimal policy of the government.

Both inflation and outright default enable the government to relax its budget constraint: Inflation generates seigniorage revenue and - to the extent that it is unexpected and bonds are nominal - reduces the real debt burden, while outright default directly reduces the outstanding debt. But both are costly, which limits the extent to which the government optimally uses them: Domestic residents hold money because of a cash-in-advance constraint on consumption, so inflation lowers their purchasing power and distorts their consumption decision. Outright default results in temporary productivity drops and bond market exclusion. The government chooses between outright default and inflation by weighing the relative costs and benefits.

The ownership and denomination structure of the debt affects the trade-offs that the government faces when making these decisions. A high share of nominal relative to real debt tilts incentives towards inflation and away from outright default. Predominantly foreign-held debt makes default more attractive since a default increases the total amount of goods available for consumption, whereas a default on domestically-held debt does not affect the aggregate resource constraint.

Equilibrium inflation and default rates are determined as a result of the interaction of inflation and default incentives with optimal debt accumulation. On the one hand, inflation and default incentives are stronger when debt is high: Lower real debt burdens are more appealing when debt is high because they enable the government to raise less revenue via distortionary labor taxes on households. On the other hand, high expected inflation and default risk raise borrowing costs and thus reduce debt accumulation in the first place. Bond portfolio characteristics that imply stronger incentives to inflate, for example, can therefore in equilibrium lead to higher or lower inflation depending on how much the government optimally borrows.

We calibrate the model to Mexico and match the observed average debt portfolio composition as well as several other key empirical moments such as the inflation rate and debt service. The model fits the data well and preserves many of the desirable features of existing sovereign default models that feature exclusively real, foreign-held debt, while also including new predictions regarding the role of the portfolio structure and inflation. Simulated data from the numerical solution of the model broadly match the empirical public finance split between tax, bond and seigniorage revenue, and predict countercyclical inflation, interest rates and net exports, as well as volatile consumption relative to GDP.

We analyze the interaction between inflation and default as policy tools to adjust the debt stock in the calibrated model. In the run-up to a default, the government uses unexpected inflation as emergency financing to reduce the real debt burden. More generally over the whole sample, default risk and inflation tend to move together, especially when cyclical conditions are bad. Inflation also tends to be high in good times when debt levels are high.

The model generates two predictions regarding the role of the portfolio structure that we highlight. First, whether a high share of nominal debt increases inflation in equilibrium depends on whether the debt is held at home or abroad. Increasing the share of foreign-held nominal debt tends to drive up inflation as the government substitutes away from default and towards the policy that is relatively more effective at reducing the debt burden. Increasing the share of domestically-held nominal debt however does not increase inflation.

Instead, the government reduces debt sufficiently for both inflation and default rates to drop in equilibrium.

Second, ownership is a key driver of incentives to accumulate debt and equilibrium debt levels. Increasing the share of debt held domestically raises total debt levels in the model. At the same time, default rates fall and inflation rates rise. The reason is that domestic debt ownership provides disincentives to default, which allows for cheaper borrowing overall and thus higher debt levels in equilibrium. This effect of ownership on debt levels is stronger than the effect of changes in denomination.

The rest of the paper is organized as follows. After a discussion of the literature, section (2) presents data on the portfolio structure of sovereign debt for a cross section of countries that motivates the paper, section (3) develops the model, section (4) discusses the calibration and main results, in section (5) we discuss the key assumptions of the model and provide some sensitivity analysis, section (6) concludes.

Literature

This paper draws on two main strands of literature. First, it builds on the quantitative sovereign default literature following Arellano (2008) and Aguiar and Gopinath (2006) who study debt problems in small open endowment economies with incomplete markets and lack of commitment. While the majority of papers in this literature focuses on exclusively real, foreign-held debt, there are a number of recent related contributions in this literature that study some of the aspects of this paper: The choice between nominal and real debt is analyzed in Engel and Park (2016) and Perez and Ottonello (2016) with no distinction between domestic and foreign creditors, and in Du et al. (2016) with no endogenous default. Du et al. (2016) show that countercyclical inflation emerges with nominal debt only if investors are risk averse. This mirrors results in Gumus (2013) who shows in a two-sector model that nominal debt (=indexed to non-tradables) reduces the countercyclicality of interest rates. Nuño and Thomas (2015) and Roettger (2014) study outright default and inflation in a model with nominal debt only. The former find that real debt is welfare improving, while the latter shows that the option to default deteriorates welfare. Na et al. (2014) analyze optimal default and exchange rate policy in an open economy with downward nominal wage rigidities and foreign-currency debt. They find that defaults are accompanied by large devaluations, and that default incentives are stronger in fixed-exchange rate economies, mirroring results on inflation in this paper. Different from all these papers, this paper studies the effects of ownership and denomination jointly, and highlights the role that their interaction plays in shaping macroeconomic outcomes.

Second, the paper draws on the literature of time-consistent public policy following Klein et al. (2008), Klein and Rios-Rull (2003) and Klein et al. (2005). It is most closely related to Martin (2009) and Diaz-Gimenez et al. (2008) both of which study optimal policies under lack of commitment in closed economies without outright default. Martin (2009) analyzes debt accumulation and inflation, highlighting the role that lack of commitment plays in driving both - a mechanism that is also at work in this paper. Diaz-Gimenez et al. (2008) analyze the role of debt denomination, showing that welfare effects are ambiguous and depend on parameters and initial conditions. We build on their results and describe conditions under which nominal debt is welfare improving in a quantitative setting.

There are other papers focusing on the interaction between monetary and fiscal policy that are related to this paper but take quite different approaches. Alfaro and Kanczuk (2010) show that nominal debt is undesirable when the choice is between labor taxes and inflation, keeping debt exogenous. We show how this result is modified when we endogenize borrowing. Niemann et al. (2013) analyze inflation dynamics in a New Keynesian model with nominal debt and find that the government inflates under discretionary policy. Arellano and Heathcote (2010) study dollarization in a model of limited enforcement, finding that it increases incentives to maintain market access and thus relaxes borrowing limits. Durdu (2009) studies GDP-indexed debt which introduces state contingency in debt payments akin to nominal debt in this paper. In her setting, an intermediate degree of indexation minimizes consumption volatility, similar to the welfare results in this paper. Indexation in her framework is exogenous while we consider optimal inflation.

Finally, nominal debt and self-fulfilling sovereign debt crises are the topic of a number of recent papers, including Aguiar et al. (2013), Rocha et al. (2013), Araujo et al. (2013) and Corsetti and Dedola (2016) among others. They highlight the that there is a trade off involved in choosing to inflate - the benefits of flexibility versus the costs of distortion - which are also present in our framework of fundamentals-driven default.[1]

On the empirical side, this paper is related to the "original sin" literature beginning with Eichengreen and Hausmann (1999) that pointed out the lack of external borrowing in local currency. Our framework can contribute to an explanation for such borrowing patterns: We find that governments can sustain positive nominal external debt, suggesting that lack of commitment is not sufficient to close nominal external sovereign debt markets. A number of other papers have studied empirical government debt portfolio composition, including Reinhart and Rogoff (2011*a*) who document the prevalence of domestic public debt, Lane and Shambaugh (2010) who study the currency composition of external debt, and a series of recent papers document increasing foreign participation in local sovereign bond markets,

[1]Several studies have explored the costs and benefits of indexed debt instruments in the context of public finance, see for instance Fischer (1975), Bohn (1990), Missale (1997) and Barro (1997), among others.

including Burger and Warnock (2007), Arslanalp and Tsuda (2014), Schreger and Du (2014) and Claessens et al. (2007).

2 Sovereign debt portfolios in the data

The key observation motivating this paper is that emerging market governments hold mixed debt portfolios both in terms of the residence of the investor base and the denomination of the debt. In this section we summarize these government debt portfolio patterns that will be incorporated in the theoretical model.

We use the database compiled by Arslanalp and Tsuda (2014).[2] It contains quarterly general government debt stock time series for 24 emerging market countries from 2004Q1 to 2015Q4 by denomination and ownership. The countries are all large emerging market borrowers included in JP Morgan's emerging market bond index and its foreign-currency bond index. The underlying data sources are wherever possible cross-country comparable data sources, in particular the IMF's quarterly external and public debt statistics databases (QEDS and QPDS), supplemented with national sources.

We focus on the ownership and denomination dimension of portfolios.[3] Debt is defined as domestic if the immediate holder of the bond is resident in the borrowing country. This is not necessarily related to the market of issuance as investors can purchase bonds listed on foreign exchanges, and we are only concerned with the residence of the bondholder in this paper. The important aspect of denomination is the degree to which the government has control over the real value of promised payment streams via inflation, so for the remainder of the paper, we will use "local currency" and "nominal" interchangeably, and similarly for "foreign currency", "indexed", and "real". Most countries issue relatively little of their debt in local currency indexed terms. Historically, the overlap between the sets of bonds that are held abroad and those that are real has been high - the conventional wisdom that emerging markets borrow from abroad in real terms.

We can represent the relevant government debt portfolio aspects in a matrix as in Table (1). Share δ of government debt is held domestically, of which α is denominated in real terms. Share $1-\delta$ is held abroad, of which κ is real.

The Arslanalp and Tsuda (2014) database contains sufficient information to calculate the debt shares from Table (1). It contains direct information on the share of debt that is owned externally, $1-\delta$. It also contains a measure of the external share in nominal debt,

[2] Available at http://www.imf.org/external/pubs/ft/wp/2014/Data/wp1439.zip

[3] This is separate from the jurisdiction under which bonds are issued. For an analysis of this aspect of sovereign bonds see, for example, Zettelmeyer et al. (2011) or Pitchford and Wright (2012).

Table 1: Sovereign debt structure

	Nominal	Real
Domestic	$\delta(1-\alpha)$	$\delta\alpha$
External	$(1-\delta)(1-\kappa)$	$(1-\delta)\kappa$

Table 2: Sovereign debt portfolios - Summary statistics

	δ	κ	α
Mean	0.67	0.61	0.18
Median	0.65	0.63	0.07
SD	0.19	0.26	0.22

$\frac{(1-\delta)(1-\kappa)}{\delta(1-\alpha)+(1-\delta)(1-\kappa)}$, and of the nominal share in total debt, $\delta(1-\alpha)+(1-\delta)(1-\kappa)$. Using these three pieces of information, we can back out δ, α, κ, the portfolio shares of interest.[4] Since there is less variation within countries over time in the portfolio breakdown than across countries, we take time-averages of portfolio shares for each country.

Table (2) shows summary statistics of the resulting portfolio shares.[5] It illustrates three main empirical regularities: Domestic debt shares are large, the majority of domestic debt is nominal, and relatively little of external debt is nominal. In other words there is some correlation between domestic debt being nominal, but it is less than perfect. Importantly, it shows that only a relatively small share of around 20% of debt is real and external ($\kappa(1-\delta)$) which is the most common assumption when studying sovereign debt problems in emerging market countries.

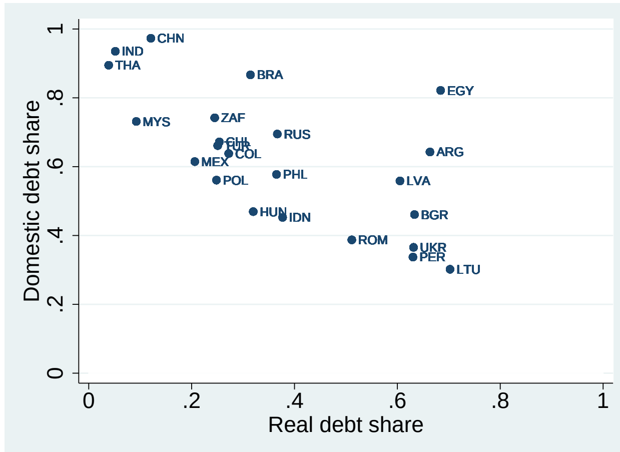
Figure (1) plots the cross-section of portfolios in the domestic-real share plane similar to Table (1). The real share is computed as the average of domestic and external real debt, weighted by the domestic and external shares.[6] The Figure shows that in terms of the quadrants from Table (1), there are relatively few countries in the top right or bottom left quadrants. The majority of sovereigns borrows largely from foreigners in real terms, or from their own citizens in nominal terms. Asia tends to occupy the top left domestic/nominal part of the graph, while Argentina is close to the bottom right external/ real corner. Overall, there is substantial heterogeneity in portfolio composition across countries.

[4]See the Appendix for details.

[5]See the Appendix for the underlying time series for each country.

[6]That is $\delta\alpha + (1-\delta)\kappa$

Figure 1: Portfolio composition in the cross-section



3 Model

We now develop a theory of sovereign borrowing, inflation and default to characterize and quantify the role that debt denomination and ownership play in shaping equilibrium outcomes. The model is a small open economy in discrete and infinite time with households, the government and international investors. Households work and save in money and government bonds to smooth consumption over time. They are subject to a cash in advance constraint on part of their consumption basket and face shocks to labor productivity. The government raises taxes, prints money and issues debt to maximize household welfare. It borrows domestically and abroad, in nominal and real terms. It lacks commitment to its policies, so that it has an incentive to borrow, inflate and default outright.

3.1 Debt structure

We assume the following debt structure. The government issues one-period claims. For every unit bond issuance, fraction δ is bought by domestic residents and fraction $1-\delta$ is bought by international investors. Fraction α of domestic holdings and fraction κ of external holdings are indexed to the domestic price level, while the remaining fractions are nominal. The government takes this structure as given, and we only need to consider total debt issuance b when solving its optimal policy problem.

For concreteness, assume that the government issues b bonds today. If the entire issuance is bought by international investors ($\delta = 0$) and is indexed to the price level ($\kappa = 1$), then b represents a promise to repay b units of consumption tomorrow to the foreign investors, just like in a standard sovereign default model with external debt only. If the bonds are nominal ($\kappa = 0$), issuing b represents a claim to b pesos tomorrow. This claim is worth b/P_t units of consumption today, but only b/P_{t+1} when repaid tomorrow, where P_t and P_{t+1} are the domestic price levels in the two periods. When inflation $\pi_{t+1} = P_{t+1}/P_t - 1$ is positive, the real value of this nominal debt will be eroded. In general, b is a promise to pay $(\kappa+\alpha)b$ units of consumption pus $((1-\kappa)+(1-\alpha))b$ pesos, split between foreigners and locals according to δ.

The assumptions underlying this debt structure are discussed in more detail in Section (5) after the main results.

3.2 Decision problems

3.2.1 Households

The representative agent maximizes the discounted expected lifetime utility from cash good consumption, credit good consumption and labor:

$$U = E_0 \sum_{t=0}^{\infty} \beta^t u(c_{1t}, c_{2t}, n_t) \tag{1}$$

where β is the subjective discount factor and we assume

$$u(c_1, c_2, n) = \gamma \frac{c_1^{1-\sigma}}{1-\sigma} + \log c_2 - \phi \frac{n^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}}$$

Preferences are separable and logarithmic in credit good consumption c_2. The Frisch elasticity of labor supply n is assumed to be constant and equal to ν. The curvature σ and weight γ on cash good consumption c_1 will determine the extent to which money and inflation are important in the economy.

Households buy cash and credit goods, as well as new money balances and, if the government is in good credit standing, bonds to carry into the next period. They finance their expenditures with labor income net of taxes, existing money balances and existing savings in government bonds. Their budget constraint is thus:

$$c_{1t} + c_{2t} + \frac{\bar{m}_{t+1}}{\bar{p}_t} + (1-d_t)\left(\frac{1-\alpha}{\bar{p}_t} + \alpha\right) q_t \delta b_{t+1} = (1-\tau_t) w_t n_t + \frac{\bar{m}_t}{\bar{p}_t} + (1-d_t)\left(\frac{1-\alpha}{\bar{p}_t} + \alpha\right) \delta b_t$$

where $\bar{m}_t$ denote money holdings, $\bar{p}_t$ is the domestic price level, b_t is the face value of government bond holdings, τ_t is the labor income tax rate, w_t the wage rate, and d_t is a binary indicator equal to 1 if the government chooses to default and 0 otherwise.[7] The within-period timing of events follows Svensson (1985): We assume that cash goods can only be purchased with existing money balances so that households are subject to cash-in-advance (CIA) constraint

$$\bar{p}_t c_{1t} \leq \bar{m}_t \tag{2}$$

This makes expected inflation costly if it constrains and thus distorts optimal cash good consumption, and makes unexpected inflation costly because the real value of existing money balances is lower than households budgeted for at the end of the previous period. Credit good purchases $\bar{p}_t c_{2t}$ can be financed with all income.

In order to make this problem stationary, we divide the nominal variables $\bar{p}_t$ and $\bar{m}_t$ as well as the nominal portion of bonds, $(1-\alpha)\delta$ by the aggregate money supply, $\bar{M}_t$, following Cooley and Hansen (1991).[8] Defining the money growth rate between today and tomorrow as $\mu_t \equiv \frac{\bar{M}_{t+1}}{\bar{M}_t} - 1$, we can then write the normalized household budget constraint as

$$c_{1t} + c_{2t} + \frac{1+\mu_t}{p_t} m_{t+1} + \left(\frac{(1-\alpha)(1+\mu_t)}{p_t} + \alpha \right) (1-d_t) q_t \delta b_{t+1} = (1-\tau_t) w_t n_t + \frac{m_t}{p_t} + \left(\frac{1-\alpha}{p_t} + \alpha \right) (1-d_t) \delta b_t \tag{3}$$

Solution to the household problem We characterize the solution to the household problem conditional on government policies. The government will solve its optimal policy problem subject to these competitive equilibrium conditions of the household. A solution to the household problem are sequences $\{c_{1t}, c_{2t}, n_t, m_{t+1}, b_{t+1}\}_{t=0}^{\infty}$ that maximize (1) subject to (2) and (3), taking as given price sequences $\{\mu_t, q_t, w_t, \tau_t, p_t\}_{t=0}^{\infty}$, a sequence of credit standings $\{d_t\}_{t=0}^{\infty}$ and initial money and bond holdings m_0 and b_0. The associated first order conditions that have to hold in every period are given by:

[7] Here d is equal to 1 both of the government has chosen to default in the current period, and if he has chosen to default in a prior period and has not yet regained market access, so his credit standing is bad. We will distinguish between the default decision and the credit standing in the recursive formulation of the government problem below.

[8] We should write bond service as $\frac{1-\alpha}{\bar{p}_t}\delta\bar{b}_t + \alpha\delta b_t$ (and analogously bond revenue). Note that after normalizing $\frac{\bar{b}}{M} = b$. For notational simplicity, we therefore go straight to b.

$$
\begin{aligned}
\frac{-u_{3t}}{u_{2t}} &= (1-\tau_t)w_t && (4)\\
u_{2t}\frac{1+\mu_t}{p_t} &= \beta E_t\left[\frac{u_{1t+1}}{p_{t+1}}\right] && (5)\\
u_{2t}\left(\frac{(1-\alpha)(1+\mu_t)}{p_t}+\alpha\right)q_t &= \beta E_t\left[(1-d_{t+1})u_{2t+1}\left(\frac{1-\alpha}{p_{t+1}}+\alpha\right)\right] && (6)\\
u_{1t}-u_{2t} &\geq 0 && (7)
\end{aligned}
$$

where expectations are taken over the productivity shock tomorrow, conditional on the realization today and u_i denotes the partial derivative of the utility functions with respect to the i^{th} argument.

The first equation is the standard intratemporal first order condition that characterizes the optimal trade-off between credit good consumption and leisure. The last condition arises from the CIA constraint. If the constraint is not binding, the condition holds with equality and consumption is not distorted - marginal utilities are equalized. Equation (6) is an Euler condition for the marginal utility of credit good consumption in periods t and $t+1$. We can rearrange it as a standard asset pricing equation to see that the bond price compensates domestic bondholders for consumption, default and inflation risk:

$$
q_t = \beta E_t\left[\underbrace{\frac{u_{2t+1}}{u_{2t}}}_{\substack{\text{Consumption}\\\text{risk}}}\underbrace{(1-d_{t+1})}_{\text{Default risk}}\underbrace{\frac{\frac{1-\alpha}{p_{t+1}}+\alpha}{\frac{(1-\alpha)(1+\mu_t)}{p_t}+\alpha}}_{\text{Inflation risk}}\right] \qquad (8)
$$

Everything else equal, a high stochastic discount factor increases household savings demand and thus the bond price. A high probability of default in the next period lowers the price and raises the interest rates that households demand in order to hold government bonds. When all bonds are nominal ($\alpha = 0$), expression (8) reduces to

$$
q_t = \beta E_t\left[\frac{u_{2t+1}}{u_{2t}}(1-d_{t+1})\frac{1}{1+\pi_{t+1}}\right]
$$

where

$$
1+\pi_{t+1} \equiv \frac{p_{t+1}(1+\mu_t)}{p_t} \qquad (9)
$$

is consumer price inflation.[9] This states that households need to be compensated to hold

[9]Recall that p is the nominal price level scaled by the aggregate money stock, that is $p \equiv \frac{\bar{p}}{\bar{M}}$, and that $1+\mu_t \equiv \frac{\bar{M}_{t+1}}{\bar{M}_t}$. Then $1+\pi_t \equiv \frac{\bar{p}_t}{\bar{p}_{t-1}} = \frac{p_t}{p_{t-1}}\frac{M_t}{M_{t-1}} = \frac{p_t(1+\mu_{t-1})}{p_{t-1}}$

bonds whose real payout is expected to be eroded through price inflation. In the solution to the optimal policy problem, the government will internalize that higher inflation and default risk reduce bond prices according to this pricing equation, which will limit its borrowing in equilibrium.

First order condition (5), finally, can shed light on how the cash in advance constraint distorts the economy. Rearranging we can write it as

$$1 = \beta E_t \left[\frac{u_{1t+1}}{u_{2t}} \frac{1}{1+\pi_{t+1}} \right]$$

This equation shows that in an undistorted steady state, the Friedman rule holds. In a steady state, when the cash in advance constraint does not bind, we have $u_1 = u_2$ from equation (7). In that case inflation is equal to β, the inverse of the real risk-free rate, and the nominal interest rate is zero. (Expected) inflation is positive, therefore, either if the cash in advance constraint binds in steady state ($u_1 > u_2$), or outside a steady state if marginal utility of cash consumption is expected to be sufficiently high ($u_{1t+1} > u_{2t}$): Households are willing to pay a positive nominal interest rate to hold money balances if they expect to be cash constrained.

3.2.2 External investors

External debt is assumed to be bought by international investors in a competitive market. Their opportunity cost is the international real risk-free rate r, and they choose government bond purchases, b_{t+1}, in order to maximize expected profits. International investors, unlike domestic residents, are risk neutral so their stochastic discount factor does not affect the pricing of the external debt. This assumption seems plausible for the frequently large, diversified, and institutional external investors that buy emerging market government debt.

Their maximization problem in each period t is

$$\max_{b_{t+1}} \Pi_t = q_{et} \left(\frac{(1-\kappa)(1+\mu_t)}{p_t} + \kappa \right) b_{t+1} - \frac{1}{1+r} E_t \left[(1-d_{t+1}) \left(\frac{1-\kappa}{p_{t+1}} + \kappa \right) \right] b_{t+1}$$

which pins downs the price of external debt as

$$q_{et} = \frac{1}{1+r} E_t \left[(1-d_{t+1}) \frac{\frac{1-\kappa}{p_{t+1}} + \kappa}{\frac{(1-\kappa)(1+\mu_t)}{p_t} + \kappa} \right] \tag{10}$$

This expression is similar to the pricing of domestic debt: Bondholders are compensated for repayment risk and, to the extent that $\kappa < 1$, inflation risk. In the case of $\kappa = 1$, the expression reduces to the standard bond pricing formula for real external sovereign debt with repayment risk only.

3.2.3 Government budget constraint

The government finances exogenous expenditures g using labor income taxes, seigniorage and net bond revenues, so we can write its budget constraint as follows:

$$
\begin{aligned}
g =& \tau_t w_t n_t + \frac{(1+\mu_t)}{p_t} - \frac{1}{p_t} \\
&+ (1-d_t)\left\{\left[\delta q_t\left(\frac{(1-\alpha)(1+\mu_t)}{p_t}+\alpha\right)+(1-\delta)q_{et}\left(\frac{(1-\kappa)(1+\mu_t)}{p_t}+\kappa\right)\right]B_{t+1}\right. \\
&\left.-\left[\delta\left(\frac{1-\alpha}{p_t}+\alpha\right)+(1-\delta)\left(\frac{1-\kappa}{p_t}+\kappa\right)\right]B_t\right\}
\end{aligned} \tag{11}
$$

where $\tau_t w_t n_t$ is labor tax revenue, $\frac{\mu_t}{p_t}$ is real seigniorage revenue, the terms on the second line are revenue from new bond issuances, and the last line captures debt repayments. The second and third lines are conditional on good credit standing, $d_t = 0$. We include labor income taxes as an empirically important source of revenue for most governments, and in order to allow the model to replicate a quantitatively realistic revenue split between bonds, seigniorage and taxes. Omitting tax revenue would exaggerate the importance of seigniorage relative to what we see in the data.

3.2.4 Production

The production side of the economy is simple with output being produced using a linear technology

$$
y_t = z_t n_t \tag{12}
$$

where z_t is an exogenous productivity shock, y_t is output and n_t labor supply. Labor will be paid its marginal product, so that in equilibrium $w_t = z_t$.

3.2.5 Market clearing

In equilibrium, money balances held by domestic households must equal money supplied by the government: $\bar{m}_t = \bar{M}_t, \forall t$, and thus normalized money supply $m_t = M_t = 1, \forall t$. Similarly for bond markets, bonds held must equal bonds supplied, $B_t = \delta b_t + (1-\delta)b_t = b_t, \forall t$. The cash in advance constraint will hold with equality in equilibrium (but not necessarily bind) since households at the margin prefer to consume rather than carry over money balances that at best have a zero return, so

$$c_{1t} = \frac{1}{p_t} \tag{13}$$

Finally the resource constraint of the small open economy, derived by combining the household and government budget constraints after imposing money and bond market clearing, will have to hold at all points in time:

$$c_{1t} + c_{2t} + g + x_t \;\; = \;\; y_t \tag{14}$$

where x_t is equal to net external savings:

$$x_t \;\; \equiv \;\; (1-\delta)\left[\left(\frac{(1-\kappa)}{p_t} + \kappa\right) b_t - \left(\frac{(1-\kappa)(1+\mu_t)}{p_t} + \kappa\right) q_{et} b_{t+1}\right] \tag{15}$$

The benefits of default are twofold in the model, as is clear from these constraints: On the one hand, it has tax smoothing benefits via the government budget constraint. A reduced debt burden allows the government to lower distortionary labor taxes and seigniorage. On the other, to the extent that debt is external, default wipes out negative net external savings and thus directly increases the resources available for consumption via the resource constraint. These two motives are typically what drives default in the literature on either domestic or external default, and here they are combined in the presence of both types of debt.

3.3 Optimal policy problem

3.3.1 Equilibrium concept

The government in the economy is benevolent but lacks commitment to its policies. This lack of commitment introduces incentives to borrow since debt issuance today is non-distortionary, and incentives to erode the real value of outstanding debt through default or inflation. We focus on time-consistent Markov perfect equilibria of the economy in which the government's optimal policies are functions of the current state of the economy only, the productivity shock z and its debt position b. When solving its problem, the government takes as given the policy functions of future governments, and so it internalizes how its choice of borrowing affects the future state of the economy and future policies. In particular, it sees that borrowing today raises default and inflation incentives in the future.

3.3.2 Sovereign default

We assume that default entails costs typically assumed in the literature: A drop in productivity and temporary market exclusion. The former captures in a simple way disruptions to the real side of the economy, the latter periods of no debt issuance observed in the data following debt crises. We abstract from debt renegotiations for simplicity and assume that debt is written off completely. Allowing for non-zero debt recovery rates would, everything else equal, reduce default incentives uniformly across bond types.[10] Importantly, the government is assumed to default simultaneously on domestic residents and foreigners without the possibility to discriminate, an assumption which we discuss in more detail in Section (5).

3.3.3 Implementability constraints

The government when solving its problem takes into account that the solution must be consistent with the competitive equilibrium conditions from the household problem. It is therefore helpful to rewrite the government budget constraint (11) in terms of allocations only using these competitive equilibrium conditions. We can substitute out the money growth rate, the tax rate and the domestic bond price using expressions (4) through (6), and substitute out the price level using (13) to get:[11]

$$u_2(c_2)\left((1-d)\left(c_1(1-\alpha)+\alpha\right)\delta b - c_2\right) - u_3(n)n = \\ (1-d)\left(\zeta_1^1(z,b') + \zeta_2^1(z,b')\delta b'\right) + d\zeta_1^0(z) \quad (16)$$

where we define the government money and domestic bond revenue functions

$$\zeta_1^1(z,b') \equiv \beta E_{z'|z}\left[(1-\boldsymbol{d}(z',b'))u_1\left(\boldsymbol{c_1^1}(z',b')\right)\boldsymbol{c_1^1}(z',b')\right. \quad (17)$$

$$\left. + \boldsymbol{d}(z',b')u_1\left(\boldsymbol{c_1^0}(z')\right)\boldsymbol{c_1^0}(z')\right] \quad (18)$$

$$\zeta_1^0(z) \equiv \beta\left(\eta\zeta_1^1(z',0) + (1-\eta)E_{z'|z}\left[u_1\left(\boldsymbol{c_1^0}(z')\right)\boldsymbol{c_1^0}(z')\right]\right) \quad (19)$$

$$\zeta_2^1(z,b') \equiv \beta E_{z'|z}\left[(1-\boldsymbol{d}(z',b'))u_2\left(\boldsymbol{c_2^1}(z',b')\right)\left(\boldsymbol{c_1^1}(z',b')(1-\alpha)+\alpha\right)\right] \quad (20)$$

Bold face letters denote equilibrium functions, superscripts denote the credit state (0 = default, 1 = repayment). u_i again denotes the derivative of the period utility function with respect to the i^{th} argument, and to simplify notation we include only the i^{th} argument, for example $u_1(\boldsymbol{c_1}(z',b'))$, since utility is separable.

We can rewrite the resource constraint (14) in a similar way as the government budget

[10] Trebesch et al. (2012) find in a comprehensive review of sovereign defaults that foreign bondholders have not received systematically unfavorable terms relative to domestic bondholders in renegotiations.

[11] See the Appendix for the derivation.

constraint. Substituting out the external bond price (10) (and using (13) one more time), the resource constraint becomes

$$c_1 + c_2 + g + (1-d)(1-\delta)\left[(c_1(1-\kappa)+\kappa)\,b - \zeta_3^1(z,b')b'\right] = zn \tag{21}$$

where we define the government external bond revenue function

$$\zeta_3^1(z,b') \equiv \frac{1}{1+r}E_{z'|z}\left[(1-\boldsymbol{d}(z',b'))\left(\boldsymbol{c_1^1}(z',b')(1-\kappa)+\kappa\right)\right] \tag{22}$$

The expressions $\zeta_1(z,b'), \zeta_2(z,b')\delta b'$ and $\zeta_3(z,b')(1-\delta)b'$ are the government's total revenue from money and bond issuance. $\zeta_1^1(z,b')$ and $\zeta_1^0(z)$ are revenue from printing money in repayment and default, respectively. $\zeta_2^1(z,b')$ is the marginal revenue from selling domestic bonds, and $\zeta_3(z,b')$ the marginal revenue from selling external bonds. Note that they are only functions of b' but not debt b.

Note that marginal revenue from external debt issuance, $\zeta_3(z,b')$, is equal to the bond price $q_e(z,b')$ which is likewise a function of b' only. This is not the case for the domestic bond price $q_d(z,b,b')$ and seigniorage revenue $\mu(z,b,b')$, both of which also depend on b because *current* marginal utility of consumption $u_2(\boldsymbol{c_2}(z,b))$ enters. Specifically, we have that $\mu(z,b,b') = \zeta_1(z,b')/u_2(\boldsymbol{c_2}(z,b))$ and $q_d(z,b,b') = \zeta_2(z,b')/u_2(\boldsymbol{c_2}(z,b))$. It is useful to focus on revenue terms that only depend on b' in order to study the borrowing incentives that the government faces which we will do below.

The expression for money revenues in default $\zeta_1^0(z)$, (19), captures that the government cannot issue new bonds, but has a η chance each period of re-entering capital markets with zero debt. The assumptions of a complete debt writedown implies that the functions in bad credit standing only depend on z but not b'.

Problem 1 (Government Problem). If the government is in good credit standing, it has the choice of whether to remain current or default:

$$V(z,b) = \max_{d\in\{0,1\}} (1-d)V^1(z,b) + dV^0(z) \tag{23}$$

In case of repayment, it chooses allocations b', c_1, c_2 and n subject to the implementability constraint, resource constraint and the household FOC for the cash in advance constraint:

$$\begin{gathered} V^1(z,b) = \max_{b',c_1,c_2,n} u(c_1,c_2,n) + \beta E_{z'|z}[V(z',b')] \\ \text{subject to (7),(16) and (21)} \end{gathered} \tag{24}$$

In case of default, it chooses c_1, c_2 and n subject to the same constraints, and re-enters capital markets with no debt in the next period with probability η:

$$V^0(z) = \max_{c_1,c_2,n} u(c_1,c_2,n) + \beta E_{z'|z}[\eta V(z',0) + (1-\eta)V^0(z')] \\ \text{subject to (7),(16) and (21)} \tag{25}$$

Definition 1 (Markov Perfect Equilibrium). *A Markov perfect equilibrium of the economy are value function $V(z,b)$ with associated policy function $\boldsymbol{d}(b,z)$, value function in repayment $V^1(z,b)$ with associated policy functions $\boldsymbol{h}(z,b)$, $\boldsymbol{c_1^1}(z,b)$, $\boldsymbol{c_2^1}(b,z)$, $\boldsymbol{n^1}(b,z)$, and value function in default $V^0(z)$ with associated policy functions $\boldsymbol{c_1^o}(z)$, $\boldsymbol{c_2^0}(z)$, and $\boldsymbol{n^0}(z)$ that solve (23), (24) and (25), with $b' = \boldsymbol{h}(z,b)$, $d = \boldsymbol{d}(z,b)$, and for (24) in good credit standing $c_1 = \boldsymbol{c_1^1}(z,b)$, $c_2 = \boldsymbol{c_2^1}(z,b,)$, $n = \boldsymbol{n^1}(z,b)$, while for (25) in bad credit standing $c_1 = \boldsymbol{c_1^0}(z)$, $c_2 = \boldsymbol{c_2^0}(z)$, $n = \boldsymbol{n^0}(z)$.*

Equilibrium prices can be read off the competitive equilibrium conditions (4) through (6), (10) and (13).

3.3.4 Characterizing the solution: An Euler equation

The solution to the optimal policy problem is characterized by an intertemporal Euler equation that describes the trade-offs that the government faces. The problem is in general non-differentiable because of the presence of default, but conditional on repaying tomorrow we can write down the first order condition for an interior borrowing choice.

Denote by λ_1 and λ_2 the Lagrange multipliers on the implementability constraint (16) and the resource constraint (21) respectively. Define the bond revenue functions conditional on repayment as follows. Let $\mathcal{Z}(b)$ be the set of z for which the government does not default, and let $\hat{\zeta}_i^1(z,b)$ be defined such that $\zeta_i^1(z,b') = E_{z'|z}[\hat{\zeta}_i^1(z',b')], i = 1,2,3$.

Then the government's Euler equation is given by:[12]

$$\lambda_1\left(\delta\left(\zeta_2^1(z,b') + b'\frac{\partial\zeta_2^1(z,b')}{\partial b'}\right) + \frac{\partial\zeta_1^1(z,b')}{\partial b'}\right) + \lambda_2(1-\delta)\left(\zeta_3^1(z,b') + b'\frac{\partial\zeta_3^1(z,b')}{\partial b'}\right) = \\ \beta E_{z'|z}\left[\left(\lambda_1'\delta\hat{\zeta}_2^1(z',b') + \lambda_2'(1-\delta)\hat{\zeta}_3^1(z',b')\right)|z' \in \mathcal{Z}(b')\right] \tag{26}$$

This equation describes how the government trades off costs and benefits of additional borrowing. On the right hand side are the marginal costs of an additional unit of bonds issued. Bonds issued today need to be repaid tomorrow which tightens the government

[12]See the Appendix for the derivation.

budget constraint and the resource constraint. This is captured by the future Lagrange multipliers for the two constraints, λ_1' and λ_2'. They are weighted by the size of the repayment, $\delta\zeta_2(z', b')$ domestically and $(1-\delta)\zeta_3(z', b')$ externally.

On the left hand side are the marginal benefits of borrowing. This marginal benefit of issuing an additional unit of debt b' is not constant, but varies with the level of b'. This can be seen by the presence of the derivatives $\frac{\partial\zeta_i^1(z,b')}{\partial b'}, i = 1, 2, 3$ on the left hand side.

To see how revenues vary with b', consider first a version of this equation when there is only external real debt and no money. This corresponds to the standard sovereign default model. In this case the only revenue from issuing b' is $\zeta_3(z, b')b' = q_e(z, b')b' = \frac{1}{1+r^*}E_{z'|z}\left[(1 - \boldsymbol{d}(z', b'))\right]$. This term exhibits the familiar Laffer curve discussed in, for example, Arellano (2008): At low levels of b', this rises with borrowing since default risk $E_{z'|z}[\boldsymbol{d}(z', b')]$ is negligible. At high levels of b', however, default risk can be sufficiently high to reduce overall revenues (that is, $\frac{\partial\zeta_3(z,b')}{\partial b'} < 0$).

In an environment with money and domestic debt, there are additional sources of revenue from money $\zeta_1(z, b')$ and domestic bonds $\zeta_2(z, b')b'$, and how these depend on borrowing is important for the incentives of the government. There is no analytical solution available for these functions so we return to a numerical example below after discussing the calibration of the model.

4 Results

4.1 Calibration

The model is calibrated and solved numerically using value function iteration. We calibrate to Mexico for reasons of data availability and because it is a large emerging market debt issuer. For all data moments except the ones related to the debt portfolio our sample starts in 1997Q1, the end of Mexico's last debt crisis. Debt portfolio data starts later, in 2004Q1. The sample ends in 2015Q2. A model period is a quarter. The parameters are summarized in Table (3). The first half of the table lists the parameters calibrated externally, the second half the parameters that are chosen in a moment matching exercise on simulated model data. Details on data sources as well as an outline of the algorithm are in the Appendix.

We choose the parameters governing the debt portfolio to replicate Mexico's average portfolio: 65% domestically owned, 78% of which in real terms, and all domestic debt in nominal terms. This implies $\delta = 0.65, \kappa = 0.78$ and $\alpha = 0$.

The probability of market re-access following a default is set to $\eta = 0.25$ implying an average exclusion period of 1 year. This is in line with estimates by Asonuma and Trebesch

Table 3: Parameters

Parameter		Value	Description/ target	Data	Model
Domestic debt share	δ	0.65	Mexico debt portfolio	65%	
Real share of external debt	κ	0.78	Mexico debt portfolio	78%	
Real share of domestic debt	α	0.00	Mexico debt portfolio	0%	
Int'l real risk-free rate	r	0.0034	3 mth US Tbill yield	1.3%	
Discount factor	β	0.98	Mexico real risk-free return	8.4%	
Re-entry probability	η	0.25	Exclusion duration	1 year	
Frisch elasticity	ν	2.00	Standard macro estimate	-	
Productivity persistence	ρ	0.95	Neumeyer and Perri (2005)	-	
Productivity volatility	σ_ϵ	0.022	Mexico GDP volatility	0.023	0.024
Labor weight	ϕ	9.640	Fraction of time working	0.33	0.33
Government spending	g	0.0363	Govt. spending / GDP	0.11	0.11
Cash good weight	γ	0.028	Cash/credit good ratio	0.81	0.82
Cash good curvature	σ	2.673	Inflation rate	0.068	0.068
Default cost	χ	-0.099	Debt service / GDP	0.050	0.053

(2016) according to which Mexico renegotiated its debt within around 1 year of defaulting, and broadly consistent with cross-country estimates by Gelos et al. (2011) according to which exclusion periods have fallen since the 1980s to an average across countries of around 2 years in the 1990s.

The international risk-free rate is set to $r = 0.0034$, which corresponds to an average annualized interest rate of 1.37% on 90-day US T-bills. We choose the disutility of labor ϕ to match a fraction of time spent working of 33%, which then determines $g = 0.0363$ to give average government spending to GDP of 11%. The Frisch elasticity of labor supply is set to $\nu = 2$, within the standard range of macro estimates.[13]

The weight of cash goods in utility γ is set to match the cash-to-credit-good ratio observed in the data in order to capture the extent to which households are exposed to monetary distortions. Let ϕ be the long term share of cash consumption in production net of national savings, $c_1 = \phi(y-g-x)$, so the targeted cash credit ratio is given by $\frac{c_1}{c_2} = \frac{\phi}{1-\phi}$. If $c_1 = M/p$, as is the case in the model in equilibrium, we can express ϕ as a function of observables: $\phi = \frac{M}{P(y-g-x)}$. Using quarterly data on nominal GDP, government spending, net exports and the M1 money stock, this implies a median cash/credit good ratio of 0.81.

In modeling the productivity process, we follow the approach Neumeyer and Perri (2005). Estimating a process for the shocks to productivity would require estimating a reliable series for Mexico's Solow residuals at quarterly frequency. To the best of our knowledge this is not

[13] Chetty et al. (2011) for example report a value of 2.84 from a meta-analysis.

possible (hours worked are not available at that frequency, for example). For this reason, we assume, like Neumeyer and Perri (2005), that the process for log productivity follows a stationary AR(1) process

$$\log z_t = \rho \log z_{t-1} + \epsilon_t, \epsilon \sim N(0, \sigma_\epsilon^2)$$

and assume the same persistence as the process estimated for the United States with $\rho = 0.95$. We then calibrate σ_ϵ so that the volatility of HP-filtered log real GDP in actual Mexican data since 1980Q1 match that generated by the simulated model. We choose a relatively longer time series here than for the rest of the calibration to avoid filtering too short a time series.

The subjective discount factor β is the inverse of the long run domestic real risk-free return. There is no real domestic asset without default or inflation risk in the model, so it is not obvious what data counterpart to use. One option would be to use financial market interest rates to back out the real risk-free rate as nominal rates less expected inflation and default risk. This approach yields negative or near zero real risk-free rates over the sample. The only nominal interest rate series available for Mexico for a long time period, the yield on 1 year Cetes bonds, is very low relative to observed inflation rates and default rates implied by EMBI spreads. An alternative approach which we therefore pursue is to use the long run return to capital observed in Mexico as a proxy for the domestic real default- and inflation-risk-free rate of return. Based on Penn World Table data this yields $\beta = 0.98$ at quarterly frequency.

The default cost is parameterized as in Arellano (2008). Productivity is reduced in default:

$$z_{def} = \min\{z, \chi\}$$

It is quantitatively important here, as well as in other studies, that the outright default costs are asymmetric and higher in times of high productivity. The asymmetry implies that the sovereign accumulates debt in good times, when he has strong disincentives to default. The curvature on cash goods in the utility is set to match inflation of 6.8% in the data. We set χ to replicate a debt service to GDP ratio of 5%.[14]

[14] Borrowing flows and debt stocks are too closely related in any one period debt model compared to the data where borrowers do not roll over their debt stocks every period. There is thus a tension whether to calibrate the model to stocks or flows. We choose flows (debt service) rather than stocks given that flow revenues and their relative contributions to the government budget are important.

4.2 Incentives and policy functions

4.2.1 Laffer curves

The government's revenue functions from the numerical solution of the model are shown in Figure (2). The Figure illustrates how default, consumption and inflation risk affect government revenues and incentives. We plot money revenue ζ_1, domestic bond revenue $\delta\zeta_2 b$, external bond revenue $(1-\delta)\zeta_3 b$ as a function of borrowing b' for a range of productivity levels z.

All three functions exhibit a Laffer curve shape that is driven by default risk: At low levels of borrowing, default risk is negligible and revenues rise with b'. For sufficiently high b', default risk is so high that revenues fall b'. Note that money revenues remain positive in default while bond revenues are zero. In all cases, lower productivity implies that the Laffer curve reaches its peak at lower levels of borrowing.

The shape of the functions below the Laffer curve peak differs: Money revenues are convex and rise strongly with borrowing at intermediate borrowing levels. Domestic bond revenue is slightly convex. External bond revenues are virtually linear.

What explains these differences in shapes? Both the domestic and external bond revenue functions are negatively affected by rising future prices since the debt portfolio considered here is partly nominal, but this effect turns out to be barely visible with external bond revenues appearing linear. Domestic bond revenue is in addition affected by household consumption risk. Expected marginal utility of consumption turns out to be high when default risk is high, and so households are relatively more willing to save in government bonds in those states of the world, which offsets some of the negative effect of default risk on revenues and explains the mild convexity of the function. Domestic bond revenue $\zeta_2(z, b')$ is thus a less sensitive function of b' than its risk-neutral counterpart $\zeta_3(z, b')$. Money revenues in turn are affected by borrowing via household money demand. Money demand increases with borrowing because households expect to be cash-constrained next period when debt is high, which increases the marginal benefit of borrowing and also partly offsets the negative effect of default risk.

4.2.2 Inflation

The shape of the money revenue curve suggests that the government uses the printing press as default risk rises. Do rising money revenues translate into higher inflation? Note that we can write inflation as $1 + \pi'(z, b, z', b') = \frac{\zeta_1(z,b')}{\mathbf{c_1}(z',b')u_2(\mathbf{c_2}(z,b))}$.[15] The effect of borrowing b' on inflation π' thus depends on how seigniorage changes relative to future cash consumption

[15] To see this, combine equations (5), (9) and (17).

Figure 2: Government revenues as a function of borrowing

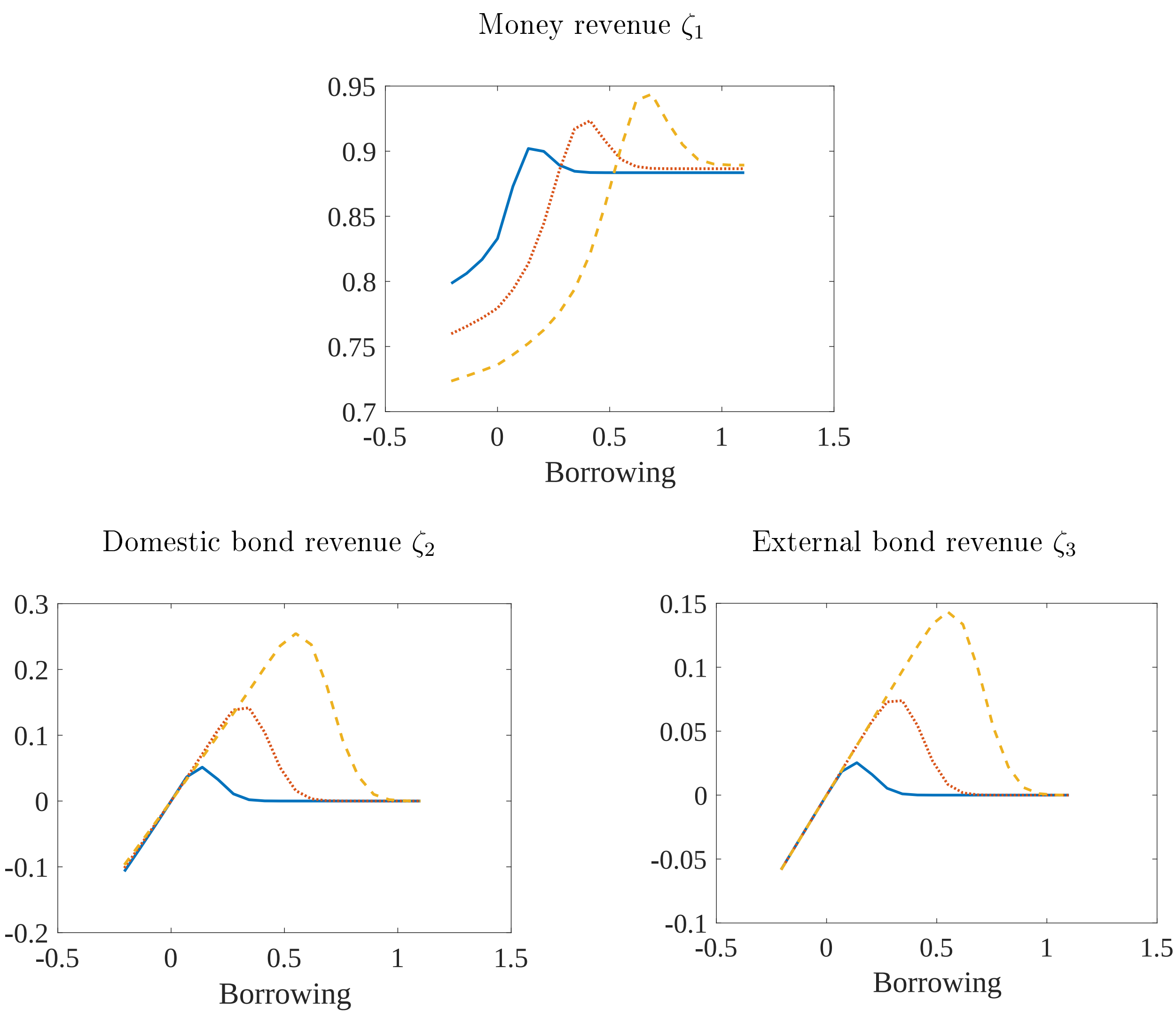
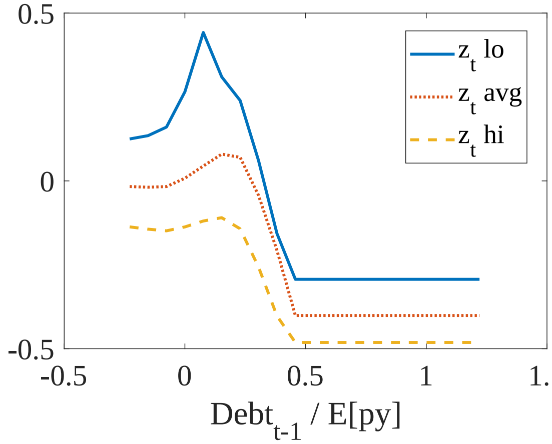


Figure 3: Realized inflation rate $\pi_t = \frac{\bar{P}_t}{\bar{P}_{t-1}} - 1$, with $z_{t-1} = E[z]$



$\boldsymbol{c_1}(z', b')$. On the one hand, if seigniorage is relatively high, agents will hold relatively more money, and so future prices and hence inflation will be relatively high. On the other hand, if seigniorage is low relative to future cash consumption demand, future prices and thus inflation will be low. For example, if z' turns out to be relatively low, implying low cash consumption $\boldsymbol{c_1}(z', b')$, we expect relatively high inflation. Note that inflation also depends on b and so while inflation can rise with borrowing, the properties of equilibrium inflation depend both on the state and on the optimal borrowing choice of the government.

Figure (3) plots equilibrium annual inflation $\pi_t \equiv \frac{\bar{P}_t}{\bar{P}_{t-1}} - 1$ as a function of debt outstanding b_{t-1} for a range of productivity realizations z_t, conditional on productivity being at its long run mean the previous period, $z_{t-1} = E[z]$, and with borrowing following the equilibrium rule $b_t = \boldsymbol{h}(z_{t-1}, b_{t-1})$.

The Figure shows that the government optimally inflates when productivity turns out to be worse than in the previous period (in this case, below average) and debt is sufficiently high but not high enough for outright default. If debt is high but productivity improved relative to yesterday and is now above trend, the government avoids very high inflation rates. The reason for this is again that its ability to raise revenue from bonds varies with productivity. In good times, bond finance is relatively cheap as well as non-distortionary, so the government uses that instead of inflation to balance its budget. In bad times it cannot raise as much revenue from borrowing due to inflation and default risk premia. As a result, it shifts its financing to inflation, as long as debt is not so high that default is imminent. The Figure suggests that the model may be able to account for countercyclical inflation finance in simulations, with inflation spiking when productivity is below average, for a given level of debt.[16]

[16] See Section (A.3) in the Appendix for graphs of other policy functions, including for interest rates and

Table 4: Model fit

	Data	Model
Tax revenue/GDP	0.11	0.11
Seigniorage/GDP	0.001	0.005
External public debt service/GDP	0.028	0.040
Domestic nominal interest rate	0.08	0.18
External real interest rate spread	0.027	0.031
Default rate	0.016	0.027
ρ(Inflation, output)	-0.25	-0.02
ρ(Domestic nominal interest rate, output)	-0.23	-0.11
ρ(External real interest rate spread, output)	-0.24	-0.56
ρ(Net exports/GDP,output)	-0.13	-0.55
ρ(Net bond revenue/GDP, output)	0.35	0.58
$\sigma(c_1 + c_2)/\sigma(y)$	1.20	1.81

4.3 Model fit

To evaluate the fit of the model to the data we simulate the model and compare statistics from model-generated and actual Mexican data. The model is simulated for 100,000 periods, the first 100 periods are discarded to eliminate the effects of initial conditions, and in both model-generated and actual data output, consumption and hours are expressed in logs and all time series are HP-filtered.[17]

Table (4) compares the model and data moments. It shows that the model replicates several aspects of the Mexican economy: The split between major sources of public finance are quite closely replicated in the model, with tax revenue accounting for 11% and seigniorage less than 1% of GDP in the model and the data. The model generates external public debt service of around 4% as a fraction of GDP, close to the data equivalent of 2.8%.

We report averages for hypothetical domestic nominal interest rates and external real interest rates in the model that we can compare to rates observed in the data. Note that these are not necessarily the same as what the model government trades since the instruments available in the model can be partially indexed (although in the case of Mexico the domestic instrument is purely nominal so it corresponds directly to the data measure). As can be seen in the Table, the model overpredicts average domestic nominal rates but matches real external spreads reasonably well. The reason that nominal rates are higher in the model than in the data is related to our calibration choice to match the real risk-free rate not based on financial market data which would have been negative. Instead we have a relatively high

government revenues.

[17]Longer simulation samples do not significantly change the results. See section (B.3) in the Appendix for a detailed description of the data sources.

real risk-free rate in the model, and correspondingly too high a nominal rate that includes inflation and default risk, relative to the data. The model does however match inflation and default rates quite well (the former because it is a calibration target, the latter is untargeted), suggesting that the default and inflation risk implied by the model are not too far off their empirical counterparts.

In terms of the co-movement of variables over the cycle, the model generates, consistent with the data, countercyclical interest rates, countercyclical net exports, procyclical overall bond revenues as well as countercyclical inflation, although the model understates the countercyclicality of inflation. In other words, the government is able to borrow more in relatively good times since that is when interest rates are low due to both countercyclical default and inflation risk. This countercyclical default risk channel is present in standard sovereign default models with exclusively real, foreign-held debt as well. Inflation also being countercyclical contributes to this as an additional factor in this model. Overall, the model preserves several of the successful features of existing models with more restrictive debt structures while providing additional predictions regarding the role of the debt structure and associated policies.

4.4 Default and inflation

The previous section showed that the government uses both default and inflation on average in the model. We now study in more detail the interaction between the two. We first consider the immediate run-up to a default, and then their co-movement more generally over the whole sample.

4.4.1 Event study

Figure (4) shows the behavior of a range of model variables in the five years prior to a typical default. The figure is based on at least 100 default samples from the simulated model data. Each line is the median over these default samples.

The Figure shows that defaults are preceded by productivity, output and consumption drops (top left panel). Output in the period of default is on average around 10% below trend. At the same time, the government loses its ability to raise revenue through both seigniorage and bond issuances, and instead relies increasingly on tax revenue (top right panel).

Against this backdrop of deteriorating macroeconomics conditions, the sovereign uses unexpected inflation in the periods before the default as "emergency financing" to reduce the debt burden and balance its budget. This can be seen in the bottom left panel where unexpected inflation rises prior to the default. *Actual* inflation remains fairly flat since

Figure 4: A typical default episode

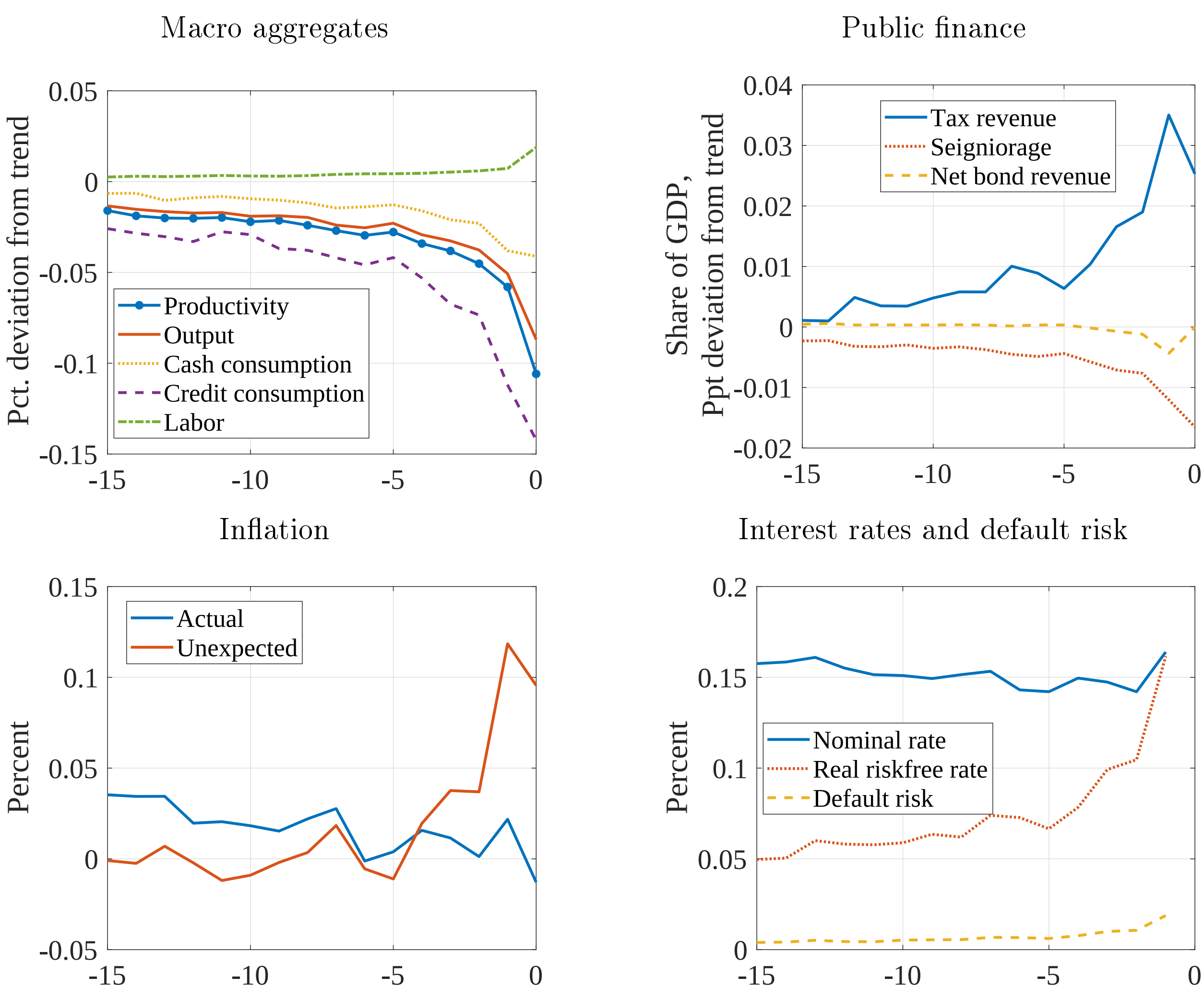


Table 5: Model outcomes conditional on cyclical conditions and default risk

		Output	Default risk	Inflation	Debt/GDP
	High default risk	-1.2%	1.4%	4.5%	3.5%
	Low default risk	1.3%	0.0%	9.9%	7.0%
Output below mean	High default risk	-1.9%	2.3%	2.2%	2.5%
	Low default risk	0.0%	0.5%	-4.1%	2.8%
Output above mean	High default risk	0.0%	0.2%	18.1%	7.4%
	Low default risk	2.0%	0.0%	11.9%	8.1%

expected inflation is low: Agents anticipate either a productivity recovery or for default to occur - in either situation there is no further reason to inflate.

In the period of the default (and during financial market exclusion), inflation drops since default is modeled relatively abstractly with full debt writedown and no other frictions that would provide incentives for the sovereign to inflate once there is no debt to service (aside from balancing the distortions arising from labor income taxes and inflation). In reality, debt crises are frequently associated with high inflation, but for reasons that are beyond the scope of this model.

The bottom right panel of the Figure finally shows that default risk and the domestic real risk-free rate increase ahead of a default, while domestic nominal rates remain almost flat. The real risk-free rate rises because of consumption risk as households expect marginal utility to be high in the case of a default. Nominal interest rates are relatively stable because inflation is expected to be low which offsets the effect of increasing consumption risk on nominal rates.

4.4.2 Co-movement

The co-movement between inflation and default risk over the whole sample depends strongly on cyclical conditions: Inflation tends to be relatively higher when *either* default risk is high *or* when output is high. Since default and output tend to move in opposite directions, this means that inflation and default risk are not unconditionally positively correlated.

Table (5) shows this in more detail. The table reports output, default risk, inflation rates and debt service to GDP ratios conditional on output and/or default risk.

The top rows of the Table show that, without conditioning on anything else, inflation tends to be low when default risk is high. This happens because periods of high default risk coincide, on average, with periods of low output in which there is less of a need to use inflation for debt financing. The middle rows of the Table show that conditional on output being below trend, however, inflation and default risk are relatively high at the same time. In other words, in situations where default is more likely to occur, inflation is used at the same time rather than as a substitute. This is also true but less relevant when output is above trend, as shown in the last two rows of the Table: Inflation is higher when default risk is, but default risk of course is negligible during relatively good times.

4.5 Debt portfolios

We next study the model predictions for how debt portfolios affect macroeconomic outcomes. We find that domestic debt tends to reduce default incentives while real debt tends to shift incentives towards outright default away from inflation. Moreover, the two forces interact: When the real share of domestic debt rises, the ownership effect dominates the denomination effect, so that debt, default rates and inflation rise.

To illustrate these effects, we solve and simulate the model for a range of portfolios and plot both Laffer curves and report simulated model statistics for each portfolio. Figure (5) plots the Laffer curves that the government faces. Each line is the sum of money and both types of bond revenues, that is $\zeta_1^1(z,b') + [\delta\zeta_2^1(z,b') + (1-\delta)\zeta_3^1(z,b')]b'$, and all are plotted for mean productivity realizations. The black circles indicate the steady state borrowing choice.

The effect of increasing domestic debt can be seen in the top left panel of the Figure which plots the Laffer curve for economies with different δ. It shows that as the domestic share δ is increased, Laffer curves remain upward sloping for wider ranges of borrowing. This reflects default risk kicking in at higher borrowing levels. The increasing share of debt that is held at home reduces default incentives and in equilibrium enables the government to extend the range of borrowing that carries no default risk. The slope of the Laffer curve is slightly convex prior to the peak. This reflects the positive contribution of revenues from money printing which increase towards the peak of the Laffer curve (recall Figure (2)).

The effect of increasing real debt can be seen in the top right panel of Figure (5). It plots the Laffer curve for varying real external shares κ and shows that real debt tends to shift incentives towards outright default. The peak of the curve is at lower levels of borrowing for higher κ: When a larger share of debt is real, the ability of the government to adjust its debt burden by means of inflation is lower, so it shifts towards outright default. The Laffer curve also becomes steeper when the real share of external debt κ is higher: For the same face value of borrowing the government generates higher revenue because marginal bond revenues are less affected by inflation risk.

These two effects interact in the bottom panel of Figure (5) which shows the effects of varying the real share of domestic debt α. Unlike in the case of higher κ, the Laffer

Figure 5: Laffer curves across portfolios

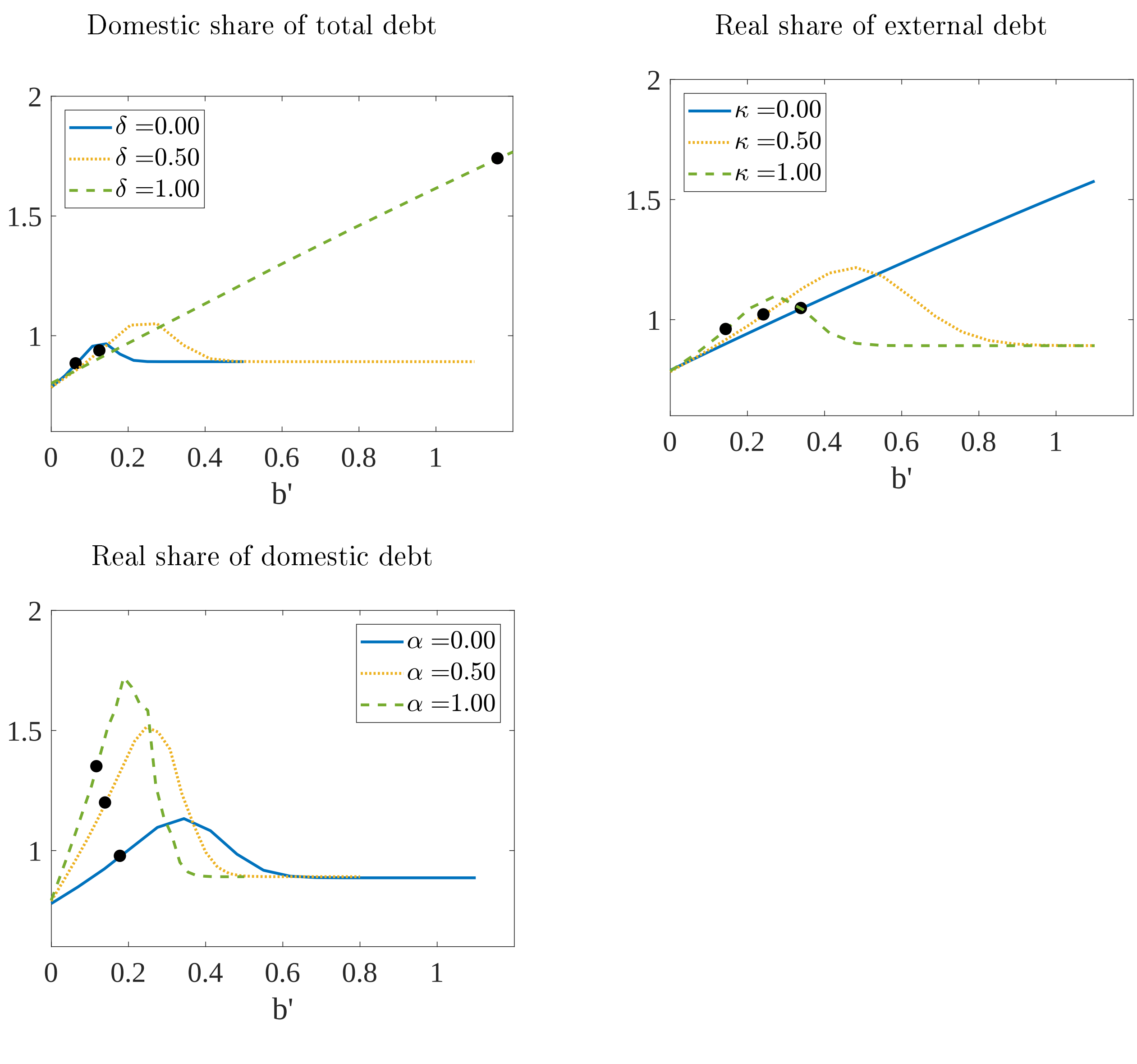


Table 6: Simulation results: The effect of denomination and ownership

		Debt/GDP	Default rate	Inflation	Tax rate
(a)	$\delta = 0.0$	0.043	0.025	0.00	0.11
	$\delta = 0.5$	0.048	0.025	0.04	0.10
	$\delta = 1.0$	0.095	0.017	0.72	0.06
(b)	$\kappa = 0.0$	0.028	0.010	0.47	0.07
	$\kappa = 0.5$	0.052	0.025	0.21	0.10
	$\kappa = 1.0$	0.052	0.025	0.00	0.11
(c)	$\alpha = 0.0$	0.053	0.026	0.07	0.11
	$\alpha = 0.5$	0.067	0.030	0.13	0.10
	$\alpha = 1.0$	0.073	0.033	0.17	0.10

curve barely shifts inwards when default becomes the more effective policy tool: More real debt, if it is held domestically, does not tilt incentives towards default as much since domestic investor bond prices are less sensitive to default risk and since default is less useful to smooth household consumption. At the same time, the curve becomes steeper as α rises: As in the case of higher external real shares, the government generates more revenue as inflation premia depress bond prices less. At the equilibrium level of borrowing, the government generates more revenue the larger the share of real domestic debt.

Table 6 compares properties from the different models in simulations. In each row, only the parameter listed in the first column is changed relative to the benchmark. Panel (a) shows that the weaker default incentives under higher domestic debt shares translate to higher debt and inflation in equilibrium: Reduced default incentives increase equilibrium borrowing and debt sufficiently for equilibrium inflation to increase overall. For sufficiently high domestic debt shares, default rates fall in equilibrium.

Panel (b) shows that with higher real shares of debt held abroad, equilibrium inflation drops and default rates rise as the government substitutes towards the more effective revenue source - outright default. The debt to GDP ratio rises moderately as bond face values are discounted less as inflation falls.

Panel (c) shows that the equilibrium effect of higher real shares of domestic debt is to drive up inflation as well as debt and default rates. Debt accumulation remains sufficiently strong as default incentives do not rise as much. As a result, the government does not shift to default sufficiently to avoid inflation as a revenue generator.

Qualitatively, these comparative statics suggest one explanation for why Mexican inflation, default risk and debt service have remained relatively stable over the last ten years despite shifts in its debt portfolio composition. Mexico's external investors increasingly lent in nominal terms with κ dropping from around 0.8 to 0.25 over the sample. This by itself puts upward pressure on inflation. At the same time, Mexico also increased the overall share of its debt held abroad: δ fell from around 0.75 to below 0.5. This by itself tends to lower inflation as default becomes relatively more appealing and debt levels lower. Taken together, these changes imply relatively stable inflation, default and borrowing incentives.

5 Sensitivity

5.1 Discussion of assumptions

The model makes a number of assumptions on the debt structure whose role we now discuss. Agents have access to partially-indexed instruments, with prices differing between externally and domestically held portions of the total debt. The assumptions that give rise to such a debt structure are that (i) the portfolio shares δ, α, κ are fixed, (iii) there is market segmentation between domestic and external investors, (iii) default is not discriminatory between domestic and external investors, and (iv) the law of one price holds so that there is no distinction between domestic inflation-indexed and foreign-currency bonds.

5.1.1 Fixed portfolio shares

One reason to assume fixed portfolio shares is technical. The model solution is a difficult fixed point problem since the bond price depends on many of the endogenous variables of the model. In a standard default model, the bond price is exclusively a function of default risk, whereas here it also depends on future prices and consumption, making convergence more difficult. Adding to this a dependence of bond prices on expected ownership and denomination of portfolios is beyond the scope of this paper.

A second reason to assume fixed portfolio shares is that is is empirically not implausible.

Table 7: Between and within volatility of portfolio shares

		Standard deviation
δ	overall	0.20
	between countries	0.19
	within countries	0.07
α	overall	0.21
	between countries	0.20
	within countries	0.08
κ	overall	0.27
	between countries	0.20
	within countries	0.19

Portfolio shares vary substantially more across than within countries. Table (7) illustrates this by comparing the standard deviation of portfolio shares between versus within groups. Specifically, the variable x_{it} is decomposed into a between groups ($\bar{x}_i = \sum_t x_{it}$) and within groups ($x_{it} - \bar{x}_i + \sum_t \sum_i x_{it}$) component.[18]

The Table shows that the cross-sectional variation in portfolio shares is more than twice as high as the within country variation for both δ and α, domestic and real domestic debt. In the case of κ, real external debt, the within variance is nearly as high as the between. The higher within variance reflects that κ has dropped in many countries over the sample, as international investors increasingly venture into domestic bond markets and buy the nominal bonds traded there. The drop in κ is in fact what some of the recent literature has focused on (for example Schreger and Du 2014). Overall, however, the table shows that variation in debt portfolio composition across countries remains more important than variation within countries over time.

5.1.2 Market segmentation

The assumption of market segmentation between domestic and foreign investors implies that there are two distinct bond prices. If there was no segmentation, then bond prices would equate by arbitrage and the model would have less to say about the ownership dimension of sovereign debt. The assumption of complete segmentation is of course stark and, just like its opposite, perfect market integration, a simplification of reality. We present some empirical evidence for limited bond market segmentation below. In the Appendix we conduct a robustness exercise that shows that an alternative model with non-segmented markets and bonds priced by foreign investors only still delivers similar properties as the benchmark model.

Empirically, markets have been segmented for the majority of sovereign debt history. Historically external public debt has been issued on foreign markets and under foreign jurisdiction, both of which impede market integration and mean that external investors had no access to domestic markets and vice versa (see for example Reinhart and Rogoff 2011*b*). More recently, debt markets have begun to become more integrated: The share of local currency debt held by foreign investors is now non-negligible, as our data show. Nonetheless, limits to market integration remain, for example due to regulatory requirements, taxes, or home bias.[19]

[18] The total number of observations is 1056, the number of groups is 22, the numbers of observations per group is 48 quarters.

[19] Efforts to improve market integration by reducing regulation etc are for example discussed in Goswami and Sharma (2011) for Asia and Jeanneau and Tovar (2006) for Latin America.

5.1.3 Non-discriminatory default

We assume non-discriminatory default since it has support in the data. Empirically, sovereign bonds are frequently structured such that default in one triggers default in another via cross-default and acceleration clauses. Choi et al. (2011) for example find that 85% of Brady issuers and 63% of other sovereigns included cross-default and acceleration clauses in their international bond issues between 1982 and 2000. Trebesch et al. (2012) document that collective action clauses (CACs), including cross-default and acceleration clauses, have become well-established market practice for bonds issued under international law. They show that Mexico in particular has since 2003 issued more than 90% of its sovereign bonds under New York law which typically includes CACs.[20]

5.1.4 Exchange rates

We abstract from the role of exchange rates as a separate policy tool. In the model there is just one price for consumption goods[21] and we assume that the law of one price holds. This renders inflation-indexed domestic currency bonds equivalent to foreign-currency bonds in the model, and external investors face the same repayment risks (inflation and default) as domestic investors.

An alternative option would be to focus on exchange rate policy and the currency distinction rather than inflation policy and the nominal/real distinction. We opt for the focus on inflation as the more relevant policy instrument since most of the countries in our sample have a flexible exchange rate regime and do not use the exchange rates as a policy instrument.

Based on IMF de-facto exchange rate regime classification, 15 out of 22 have floating exchange rates. The exceptions are Bulgaria, Latvia, Lithuania and Malaysia which have currency pegs or boards, China and Argentina which have crawls, and Russia. Latvia adopted the Euro at the beginning of 2014, Lithuania at the beginning of 2015 (our sample ends at the end of 2015). Russia, classified as "Other" in the IMF data, is classified as having a crawling peg or band by Ilzetzki et al. (2017).

5.2 Robustness: Alternative parameterizations

We investigate the effects on the benchmark results of changes in the key parameters of the model that drive default and inflation: the default cost parameter χ and the curvature

[20] Other support for the prevalence of simultaneous default on domestic and foreign bondholders includes Erce and Diaz-Cassou (2010) who show that seven out of eleven external restructurings in a sample of recent debt crises were preceded or followed by domestic debt restructurings.

[21] There are two consumption goods, credit and cash goods, but they have the same price, see Lucas and Stokey (1987)

Table 8: Robustness

	Benchmark	Default cost			Inflation cost		
	$\chi = -0.099$ $\sigma = 2.673$	$\chi = 0.10$	$\chi = 0.00$	$\chi = -0.15$	$\sigma = 2.00$	$\sigma = 3.50$	$\sigma = 5.50$
Inflation	0.068	-0.075	-0.053	0.192	0.045	0.097	0.128
Default rate	0.027	0.027	0.063	0.012	0.018	0.031	0.040
Tax rate	0.11	0.12	0.12	0.10	0.13	0.08	0.05
Debt/GDP	0.053	0.000	0.008	0.096	0.046	0.058	0.062
Cash/credit goods	0.81	0.84	0.84	0.81	0.40	1.39	2.69
Hours worked	0.33	0.33	0.33	0.33	0.28	0.40	0.53

of cash good consumption σ. Both affect the relative cost of default versus inflation. The default cost parameter affects it directly, a lower value of χ means productivity costs of default apply already at a lower threshold for productivity. Inflation becomes costlier as the intertemporal elasticity of substitution (IES) of the cash good falls - that is as σ rises.

Table (8) compares simulated model statistics under the different parameterizations. The first half of the Table shows model outcomes for a range of default costs χ. It shows that sufficiently cheap default - high χ - raises default incentives so much that in equilibrium next to no debt is sustainable. This implies low default and inflation rates as there are nearly no financing needs. As default becomes more expensive, more debt becomes sustainable, and default and inflation rates rise. For very high default costs, equilibrium default rates fall again, despite debt - and thus inflation - continuing to rise.

The second half of the Table shows the effects of changes in the cost of inflation. The costlier inflation, that is the higher σ, the stronger the disincentives to use inflation finance and thus the higher the need and ability to issue debt. In equilibrium this leads to debt, inflation, and default rates being relatively high when σ is high. Unlike in the case of the default cost, the effects are monotone here: The initial disincentive to inflate due to a lower IES is dominated by the increased incentive to inflate because of debt accumulation for the range of σ considered. Cash good consumption is lower the higher σ to maintain the households' optimal trade-off between cash and credit goods.[22]

[22] We could also depart from the assumption of log utility for credit goods, but this affects outcomes qualitatively in the same way as changes in the IES of cash goods: A drop in the IES of credit goods relative to the benchmark makes credit good fluctuations relatively costly and thus cash good fluctuations relatively cheap, so it raises inflation incentives. In equilibrium, this lowers debt and inflation rates, and raises credit good consumption. These effects are the same as an increase in the IES of cash goods, what matters is the relative IES between the two goods.

6 Conclusion

This paper has developed a model of sovereign borrowing and default with mixed government debt portfolios along the dimensions of denomination and ownership. In a calibrated version of the model the government optimally uses both default and inflation to adjust its debt burden and smooth household consumption. Unexpected inflation rises in the run-up to a default, and both inflation and default risk tend to be high during cyclical downturns. The structure of the debt portfolio affects equilibrium outcomes: Increasingly nominal debt raises inflation only if it is held abroad, and domestic debt ownership is a key driver of debt accumulation because default incentives at home are weaker.

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Appendix
For Online Publication

A Model Appendix

A.1 Deriving the implementability constraint

Write the government budget constraint (11) recursively as

$$\frac{1}{p}+(1-d)\left[\delta\left(\frac{1-\alpha}{p}+\alpha\right)\right]b+g-\tau wn+x=\frac{1+\mu}{p}+(1-d)\left[\delta q\left(\frac{(1-\alpha)(1+\mu)}{p}+\alpha\right)\right]b'$$

where we have imposed money market clearing and x is defined as in (15). Then substitute for μ and q using (5) and (6), and use the definitions of ζ_1^1 and ζ_2^1 from the main text to get

$$u_2\left[\frac{1}{p}+(1-d)\left[\delta\left(\frac{1-\alpha}{p}+\alpha\right)\right]b+g-\tau wn+x\right]=(1-d)\left(\zeta_1^1(z,b')+\left[\delta\zeta_2(z,b')\right]b'\right)+d\zeta_1^0(z)$$

Now substitute for the tax rate using (4)

$$u_2\left[\frac{1}{p}+(1-d)\left[\delta\left(\frac{1-\alpha}{p}+\alpha\right)\right]b+g-y+x\right]-u_3n \quad = \quad (1-d)\left(\zeta_1^1(z,b')+\left[\delta\zeta_2(z,b')\right]b'\right)+d\zeta_1^0(z)$$

ans use market clearing (14) and 13 to arrive at (16). This constraint is the same as in a closed economy (except that the debt stock and borrowing are scaled by δ and that x can be $\neq 0$), external savings are defined via the resource constraint.

A.2 Deriving the government Euler condition

Recall that we denote by λ_1 and λ_2 the Lagrange multipliers on the implementability constraint (16) and the resource constraint (21), let $\mathcal{Z}(b)$ be the set of z for which the government does not default given b, and let $\hat{\zeta}_i^1(z,b)$ be defined such that $\zeta_i^1(z,b')=E_{z'|z}[\hat{\zeta}_i^1(z',b')], i=1,2,3$.

FOC with respect to b':

$$\lambda_1\left(\frac{\partial\zeta_1^1(z,b')}{\partial b'}+\delta\zeta_2^1(z,b')+\delta b'\frac{\partial\zeta_2^1(z,b')}{\partial b'}\right)+$$
$$\lambda_2(1-\delta)\left(\zeta_3^1(z,b')+b'\frac{\partial\zeta_3^1(z,b')}{\partial b'}\right)+\beta E_{z'|z}\left[\frac{\partial V^1(z,b')}{\partial b'}|z'\in\mathcal{Z}(b')\right]=0$$

Envelope condition:

$$\frac{\partial V^1(z,b)}{\partial b} = -\lambda_1 \delta \hat{\zeta}_2^1(z,b) - \lambda_2(1-\delta)\hat{\zeta}_3^1(z,b)$$

Combining these by substituting out the value function yields expressions (26).

A.3 Equilibrium policy functions

Figure (6) plots equilibrium policies as a function of the state: Labor income tax rates, the domestic bond yield (equivalently, the nominal interest rate since the share of real domestic debt is zero in the benchmark calibration), expected inflation, the money growth rate and the government's three sources of revenue as a share of spending, labor income taxes, seigniorage and net borrowing. The rates are all annualized. All objects depend not just on the state but also on the borrowing decisions b', and we have imposed that the optimal borrowing policy is being followed, that is $b' = \boldsymbol{h}(z,b)$.

A.4 A model without market segmentation

We consider a version of the model with one modification: We assume that bond prices across markets do not differ but that instead all debt is priced by the international investors. We still treat the observed portfolio shares as exogenous and keep them the same as in the benchmark model. This modification amounts to a relaxation of the segmented market assumption. In this setting $q = q_e$, and this is the only change to the model. The implementability constraint can now be derived as

$$u_2\left[\frac{1}{p} + (1-d)\left[\delta\left(\frac{1-\alpha}{p}+\alpha\right)\right] b + g - y + x\right] - u_3 n$$
$$= (1-d)\left(\zeta_1^1(z,b') + \zeta_3(z,b')\frac{(1-\alpha)\zeta_1(z,b')+\alpha u_2}{(1-\kappa)\zeta_1(z,b')+\kappa u_2}\delta b'\right) + d\zeta_1^0(z) \quad (27)$$

where domestic marginal revenue ζ_2 is replaced by $\zeta_3(z,b')\frac{(1-\alpha)\zeta_1(z,b')+\alpha u_2}{(1-\kappa)\zeta_1(z,b')+\kappa u_2}$.

We solve and simulate this modified model under the benchmark calibration, and find that the results are very similar. Figure (7) compares the total Laffer curve of the government with and without market segmentation.[23] The differences are visually small.

Table (9) compares key simulated statistics from the two models. Again, the differences are negligible. Note that what is of course different is the bond price for the domestic asset

[23]That is, the sum of money, external and domestic bond revenue as a function of borrowing: $\zeta_1^1(z,b') + \zeta_3(z,b')\frac{(1-\alpha)\zeta_1(z,b')+\alpha u_2}{(1-\kappa)\zeta_1(z,b')+\kappa u_2}\delta + \zeta_3(z,b')(1-\delta)$

Figure 6: Equilibrium policy functions

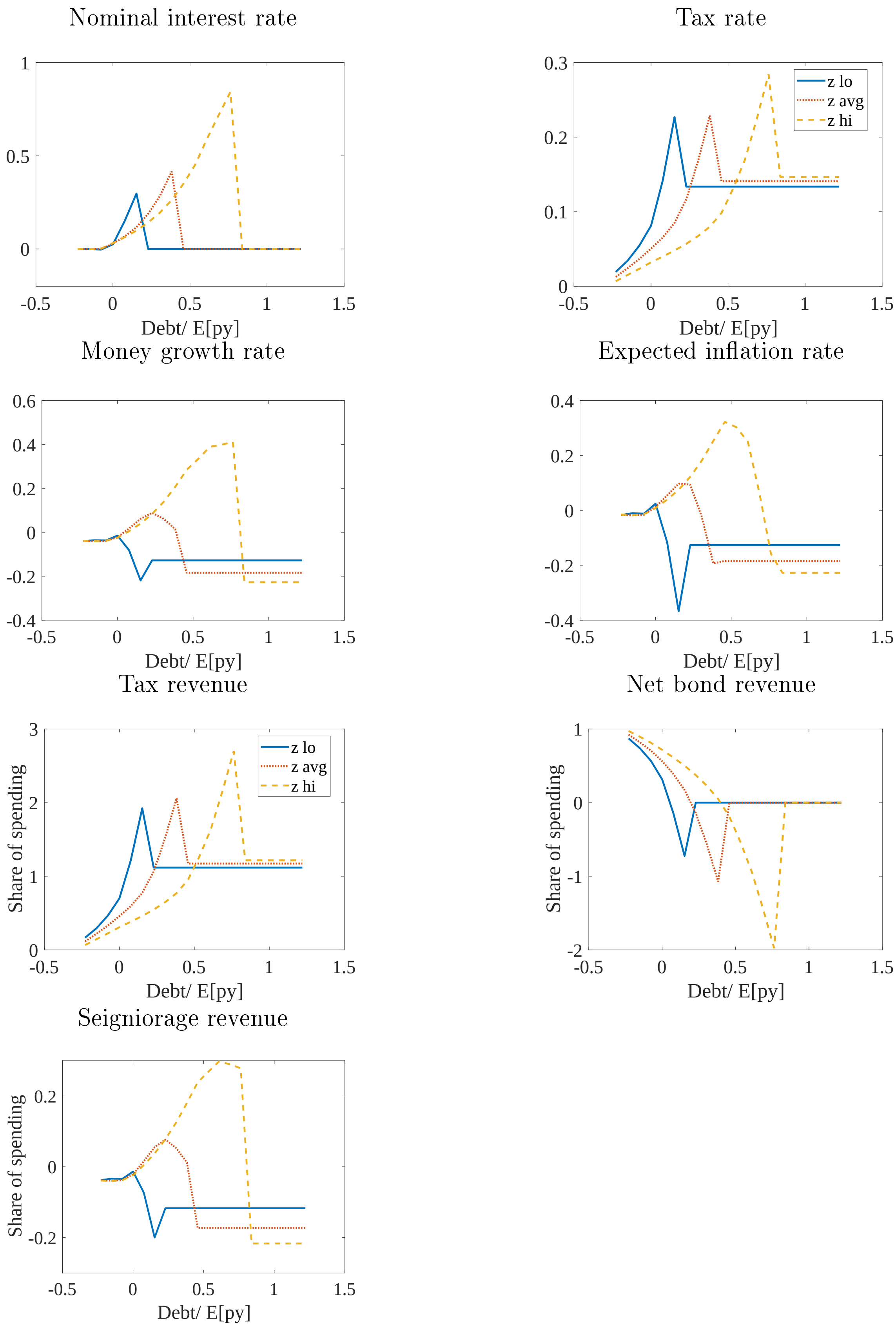

Figure 7: Laffer curves without market segmentation

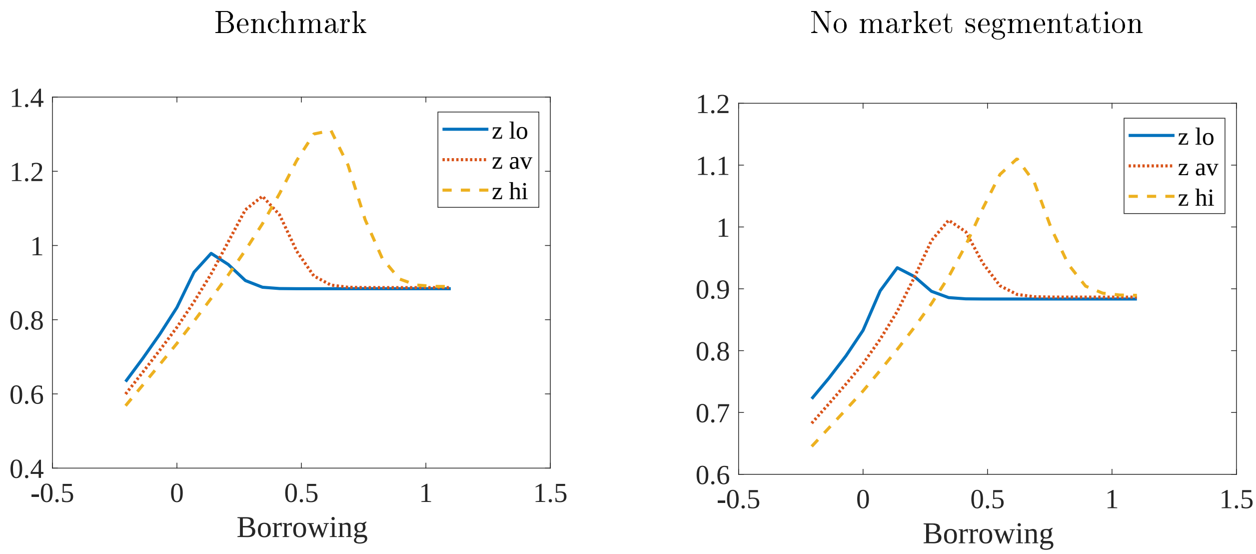

Table 9: Model statistics without market segmentation

	Benchmark	No market segmentation
Inflation rate	0.068	0.066
Debt service/ GDP	0.053	0.052
Cash/ credit good ratio	0.82	0.82
GDP volatility	0.024	0.024
Domestic nominal interest rate	0.18	0.18
External real interest rate spread	0.031	0.031
Tax revenue/GDP	0.11	0.11
Seigniorage/GDP	0.005	0.005
External public debt service/GDP	0.040	0.040
ρ(Inflation, output)	-0.02	-0.01
ρ(Domestic nominal interest rate, output)	-0.11	-0.11
ρ(External spread, output)	-0.56	-0.56
ρ(Net bond revenue/GDP, output)	0.58	0.58
Default rate	0.027	0.027

in the model, but this does not affect either government incentives or aggregate outcomes very much. This is consistent with the domestic and external bond revenue functions in the benchmark model being fairly similar as well and the curvature introduced by household risk aversion not being a dominant factor shaping it (recall Figure (5) in the main text).

A.5 Algorithm

We solve the model by value function iteration in the following steps:

1. For each z, b, compute an initial guesses for value functions $V(z,b), V^0(z,b), V^1(z,b)$, consumption function $\boldsymbol{c_2}(z,b)$, labor $\boldsymbol{n}(z,b)$ and default function $\boldsymbol{d}(z,b)$ by solving the final period solution of the finite horizon version of the model with cash consumption $\boldsymbol{c_1}(z,b)$ fixed at an arbitrary level, $b' = \boldsymbol{h}(z,b) = 0$ and $\zeta_i(z,b) = 0, i = 1,2,3$.

2. For each z, b, given continuation values $V(z,b), V^0(z,b), V^1(z,b)$, consumption functions $\boldsymbol{c_2}(z,b), \boldsymbol{c_1}(z,b)$ and default function $\boldsymbol{d}(z,b)$, solve the problem of the government in equations (23) through (25).

3. Compare new value functions, consumption functions and the default function with the previous iteration. If the distance is small enough, stop, otherwise continue with Step 2.

Expectations are computed with numerical integration. The government problem is solved by assuming and verifying that the inequality constraints binds in all states.

B Data Appendix

B.1 Portfolio data

The data set is available at http://www.imf.org/external/pubs/ft/wp/2014/Data/wp1439.zip. We use the following variables with corresponding sheet and row in the data set spreadsheet:

- (GGall) General government debt: Table 1, 1:25
- (ExtGGall) External general government debt: Table 2, 1:25
- (GG) General government debt securities: Table 1, 27:52
- (ExtGG) External general government debt securities: Table 2, 53:78
- (LCCG) Local currency central government debt securities: Table 1, 54:79

- (ExtLCCG) External local currency central government debt securities: Table 2, 107:132
- (GGy) General government debt to GDP: FX, 27:51

Foreign holdings of general government debt exclude foreign official loans. Data on foreign ownership of local-currency central government debt securities are available for most countries in the sample from national data sources. Like Arslanalp and Tsuda (2014), we use this as a proxy for foreign ownership of local-currency general government debt securities, assuming that foreign holdings of local government debt securities are small. Egypt and Ukraine are dropped due to their near-crisis status throughout the sample.

The authors do not discuss how local currency indexed debt is classified. Assuming that it is treated as local currency debt, our estimates for α and κ are lower bounds. Schreger and Du (2014) who construct similar measures of government debt portfolio shares note that, where available, nonresident holdings holdings of indexed debt are very small relative to nonresident holdings of local currency debt, suggesting that our estimate of κ is unlikely to be a substantial underestimate. Similarly for domestic real debt shares, indexed bond markets in general tend to be small relative to non-indexed so α is unlikely to be large and certainly smaller than κ.

B.2 Inferring portfolio shares

Consider the following stylized representation of the sovereign's debt portfolio:

	Nominal	Real	Nom+Real
Domestic	$\delta(1-\alpha)$	$\delta\alpha$	δ
External	$(1-\delta)(1-\kappa)$	$(1-\delta)\kappa$	$1-\delta$
Int+Ext	x	$1-x$	1

We can obtain estimates of δ, x and the external share of nominal debt $y \equiv \frac{(1-\delta)(1-\kappa)}{x}$ from the database as follows. The domestic debt share is computed as $\delta = 1 - \frac{ExtGGall}{GGall}$. The nominal share of debt is approximated as $x = \frac{LCCG}{GG}$. Ideally we would like to use total central government debt securities in the denominator. While these are not available in the publicly available database, the authors have kindly shared those data with me, and the results are virtually unchanged, so we continue to report the results based on the published data. x is a good approximation as long as local government debt is small relative to general government debt which tends to be true in the data in most countries. The external share of nominal debt is calculated as $y = \frac{ExtLCCG}{LCCG}$.

We then use these estimates to infer α and κ: Using the definition of y,

$$\kappa = 1 - \frac{xy}{1-\delta}$$

and since from the table, $1 - \alpha = \frac{x-(1-\delta)(1-\kappa)}{\delta}$,

$$\alpha = 1 - \frac{x(1-y)}{\delta}$$

B.3 Data sources for calibration and model fit

To calibrate and compare the model to the data we use the following data.

Real and nominal GDP, consumption and government spending, nominal net exports, the M1 money stock, the CPI index and the nominal interest rate are from the OECD, quarterly and seasonally adjusted, 1997Q1 - 2015Q2, except for the inters rate which starts in 2001Q1. The external real interest rate spread is the JP Morgan EMBI from 1997Q1 through 2015Q2.

Tax revenue and the M0 monetary base are from Banco de Mexico, quarterly seasonally adjusted 1997Q1 through 2015Q2. Tax revenue excludes consumption and excise taxes.

Net bond revenue is calculated residually from the government budget constraint: Tax revenue plus seigniorage less government spending.

The return to capital is calculated using annual Penn World Table data on output, the capital stock, the labor share and the investment to output ratio. We compute it as $r = \frac{\gamma}{K/Y} - \delta$ where γ is the average capital income share, $\delta = \frac{I}{K}$ is the depreciation rate and $\frac{K}{Y}$ is the capital stock to income ratio.

The empirical default frequency is based on Reinhart and Rogoff (2011b) since 1945.

B.4 Portfolio share time series

Figure 8: Debt portfolio structure: Domestic share of total debt δ

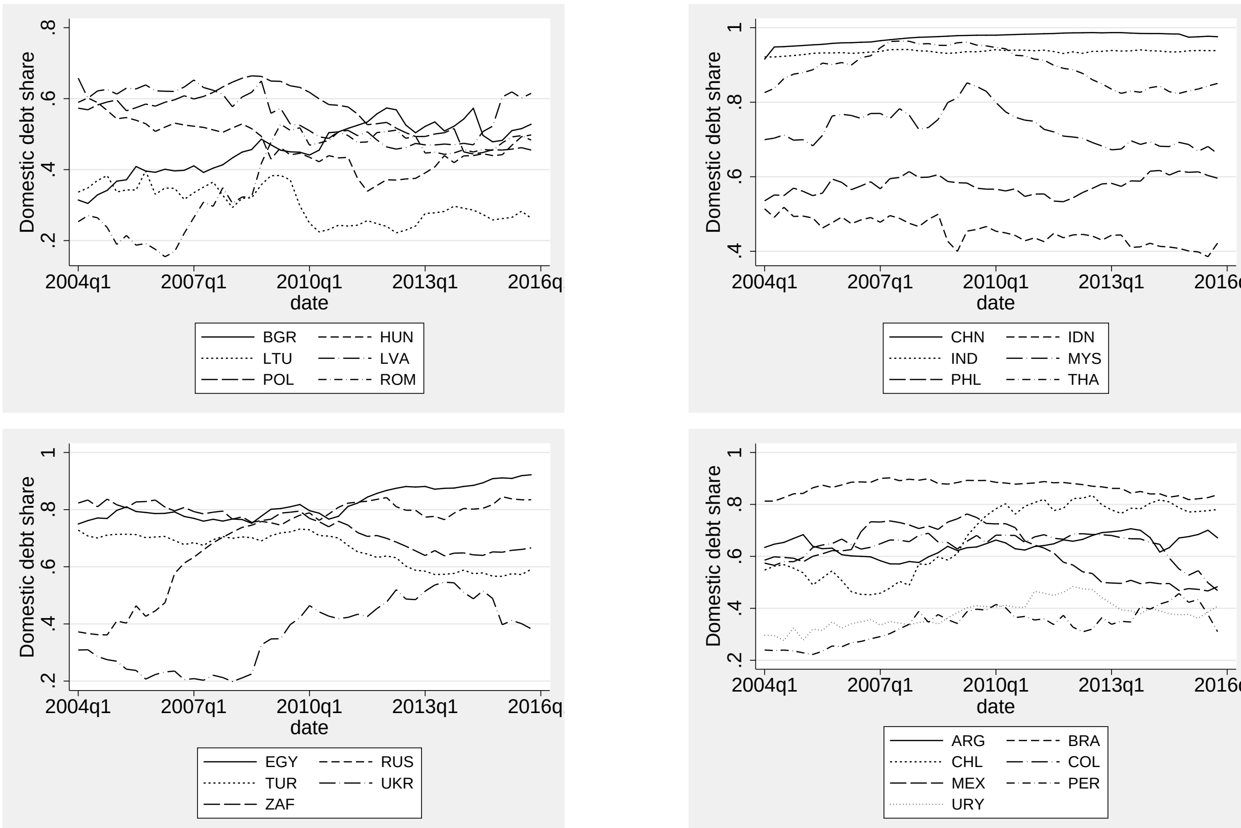

Figure 9: Debt portfolio structure: Real share of external debt κ

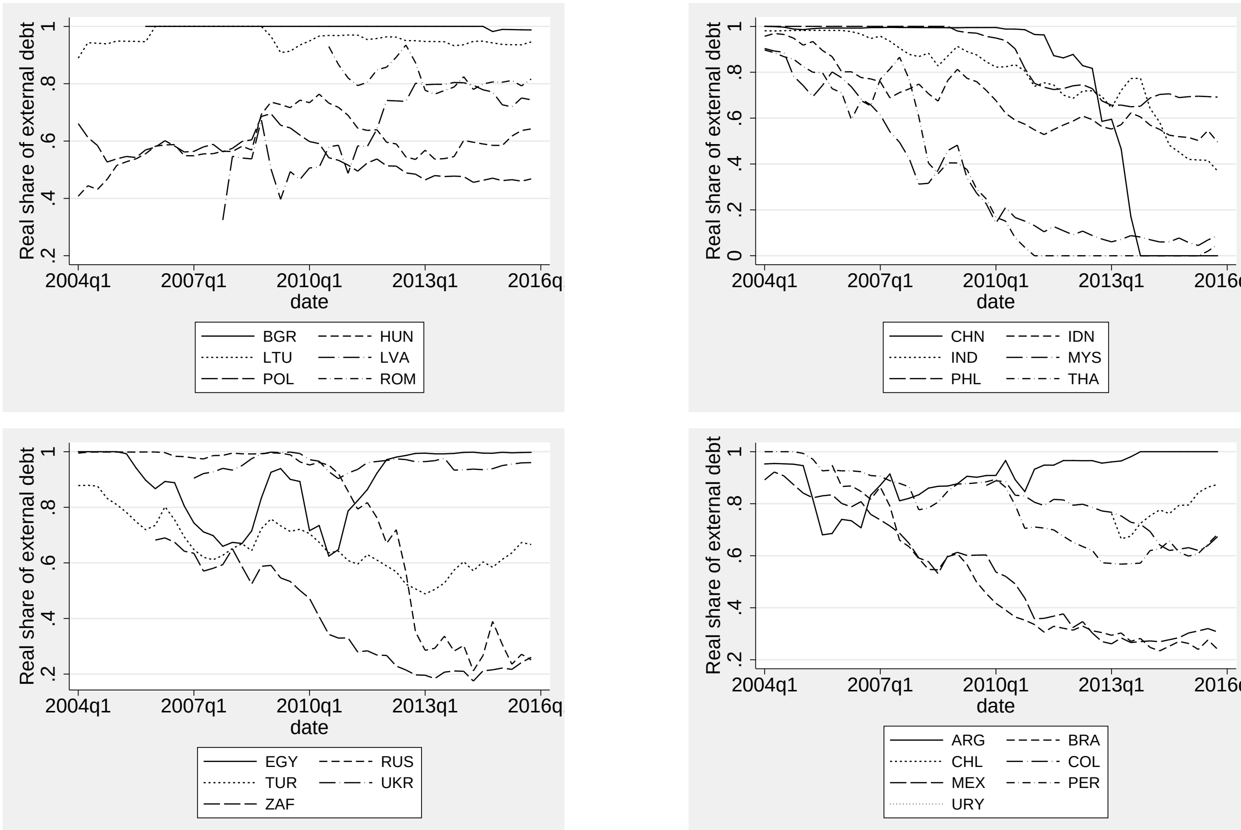

Figure 10: Debt portfolio structure: Real share of domestic debt α